Answer Set Solving in Practice

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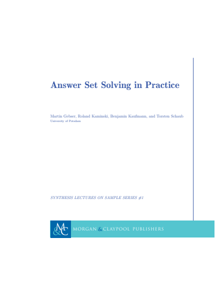
Rough Roadmap

Resources

- Course material
  - `http://potassco.sourceforge.net/teaching.html`
  - `http://moodle.cs.uni-potsdam.de`
  - `http://www.cs.uni-potsdam.de/wv/lehre`
- Systems
  - clasp `http://potassco.sourceforge.net`
  - dlv `http://www.dlvsystem.com`
  - smodels `http://www.tcs.hut.fi/Software/smodels`
  - gringo `http://potassco.sourceforge.net`
  - lparse `http://www.tcs.hut.fi/Software/smodels`
  - clingo `http://potassco.sourceforge.net`
  - iclingo `http://potassco.sourceforge.net`
  - oclingo `http://potassco.sourceforge.net`
  - asparagus `http://asparagus.cs.uni-potsdam.de`

The Potassco Book

1. Motivation
2. Introduction
3. Basic modeling
4. Grounding
5. Characterizations
6. Solving
7. Systems
8. Advanced modeling
9. Conclusions



Resources

- http://potassco.sourceforge.net/book.html
- http://potassco.sourceforge.net/teaching.html

Literature

Books [4], [29], [53]

Surveys [50], [2], [39], [21], [11]

Articles [41], [42], [6], [61], [54], [49], [40], etc.

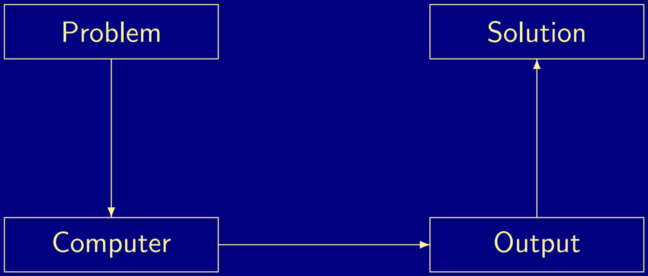
Motivation: Overview

Outline

Potassco

Informatics

"What is the problem?" versus *"How to solve the problem?"*



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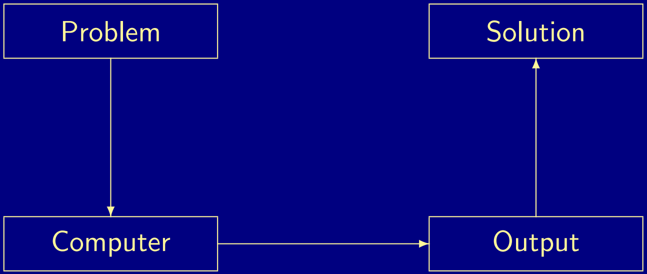

Traditional programming

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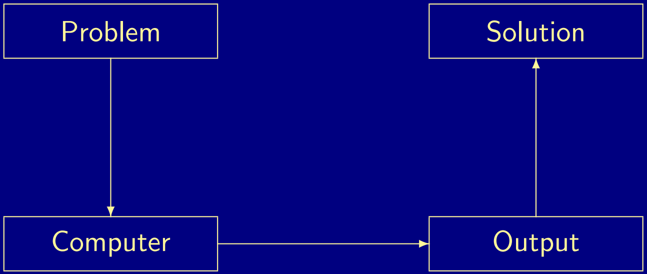

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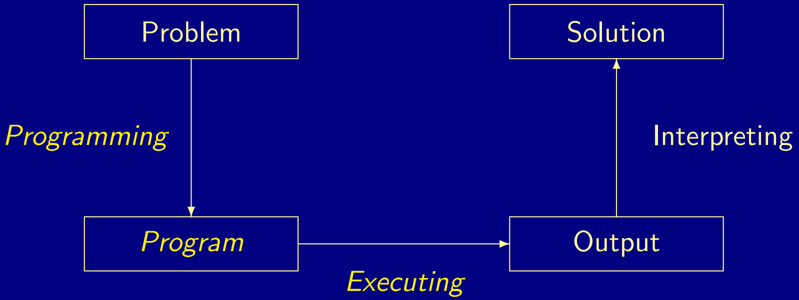

Declarative problem solving

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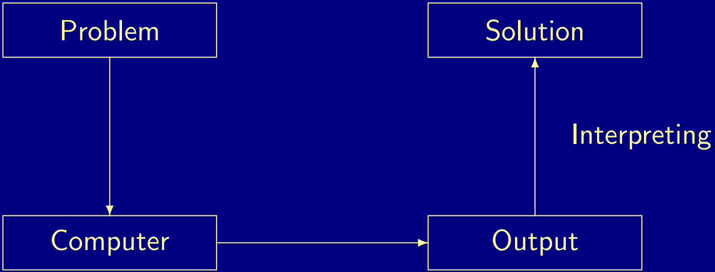

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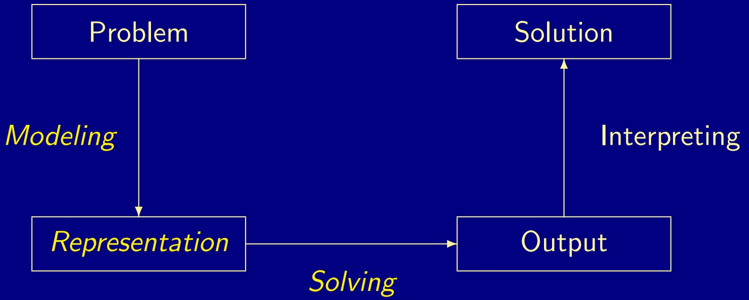

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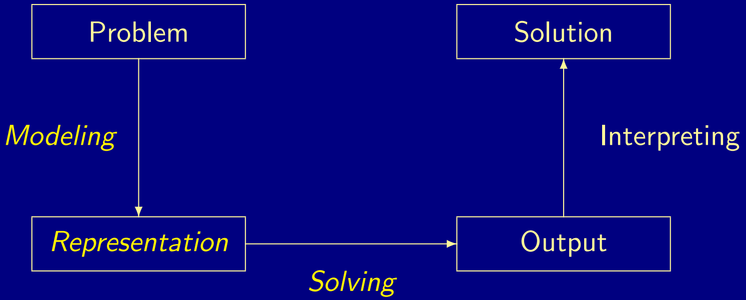

Outline

Answer Set Programming

in a Nutshell

- ASP is an approach to declarative problem solving, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
- ASP has its roots in
  - (deductive) databases
  - logic programming (with negation)
  - (logic-based) knowledge representation and (nonmonotonic) reasoning
  - constraint solving (in particular, SATisfiability testing)
- ASP allows for solving all search problems in NP (and NP^{NP}) in a uniform way
- ASP is versatile as reflected by the ASP solver *clasp*, winning first places at ASP, CASC, MISC, PB, and SAT competitions
- ASP embraces many emerging application areas

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in a Hazelnutshell

- ASP is an approach to declarative problem solving, combining
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  tailored to Knowledge Representation and Reasoning

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ASP = DB+LP+KR+SAT

Outline

KR's shift of paradigm

Theorem Proving based approach (eg. Prolog)

1. Provide a representation of the problem
2. A solution is given by a derivation of a query

Model Generation based approach (eg. SATisfiability testing)

1. Provide a representation of the problem
2. A solution is given by a model of the representation

Automated planning, Kautz and Selman (ECAI'92)

Represent planning problems as propositional theories so that models not proofs describe solutions

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Model Generation based Problem Solving

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constraint satisfaction problem	assignment
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first-order theories	Herbrand models
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default theories	extensions
⋮	⋮

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Potassco

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LP-style playing with blocks

Prolog program

```
on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
```

Prolog queries

```
?- above(a,c).
true.

?- above(c,a).
no.
```

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Prolog queries (testing entailment)

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?- above(a,c).
true.

?- above(c,a).
no.
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LP-style playing with blocks

Shuffled Prolog program

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Prolog queries

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?- above(a,c).

Fatal Error: local stack overflow.
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Prolog queries (answered via fixed execution)

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SAT-style playing with blocks

Formula

$$
\begin{array}{ll}
 & on(a,b) \\
\land & on(b,c) \\
\land & (on(X,Y) \rightarrow above(X,Y)) \\
\land & (on(X,Z) \land above(Z,Y) \rightarrow above(X,Y))
\end{array}
$$

Herbrand model

$$
\left\{
\begin{array}{rrrrr}
on(a,b), & on(b,c), & on(a,c), & on(b,b), & \\
above(a,b), & above(b,c), & above(a,c), & above(b,b), & above(c,b)
\end{array}
\right\}
$$

SAT-style playing with blocks

Formula

$$
\begin{array}{ll}
 & on(a,b) \\
\wedge & on(b,c) \\
\wedge & (on(X,Y) \rightarrow above(X,Y)) \\
\wedge & (on(X,Z) \wedge above(Z,Y) \rightarrow above(X,Y))
\end{array}
$$

Herbrand model

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Herbrand model (among 426!)

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Potassco

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➥ Answer Set Programming (ASP)

Potassco

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Answer Set Programming *at large*

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Potassco

Answer Set Programming *commonly*

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Answer Set Programming *in practice*

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Stable Herbrand model

$$\{ \texttt{on}(\texttt{a},\texttt{b}),\ \texttt{on}(\texttt{b},\texttt{c}),\ \texttt{above}(\texttt{b},\texttt{c}),\ \texttt{above}(\texttt{a},\texttt{b}),\ \texttt{above}(\texttt{a},\texttt{c}) \}$$

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Stable Herbrand model (and no others)

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ASP versus LP

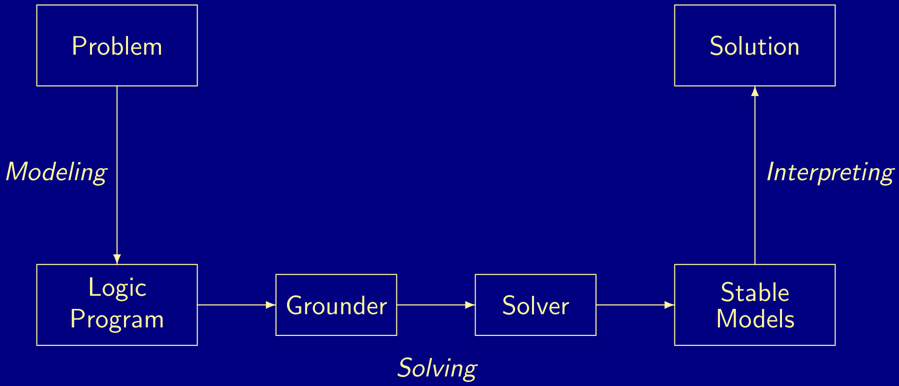
ASP	Prolog
Model generation	Query orientation
Bottom-up	Top-down
Modeling language	Programming language
Rule-based format	
Instantiation Flat terms	Unification Nested terms
(Turing +) $NP(^{NP})$	Turing

ASP versus SAT

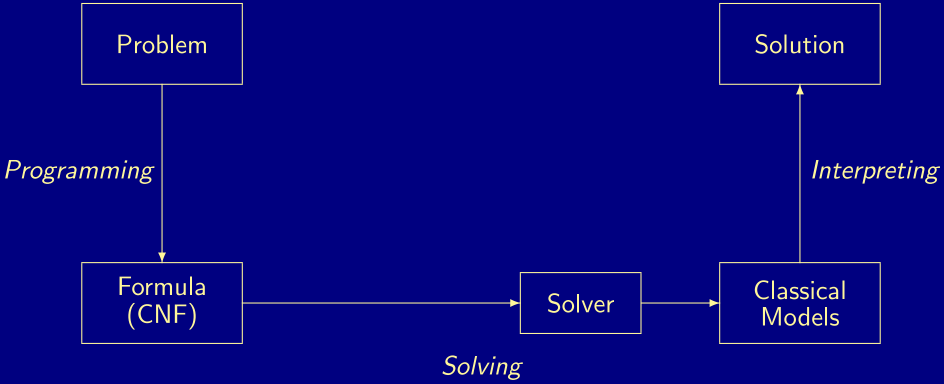
ASP	SAT
Model generation	
Bottom-up	
Constructive Logic	Classical Logic
Closed (and open) world reasoning	Open world reasoning
Modeling language	—
Complex reasoning modes	Satisfiability testing
Satisfiability	Satisfiability
Enumeration/Projection	—
Intersection/Union	—
Optimization	—
(Turing +) $NP(^{NP})$	NP

Outline

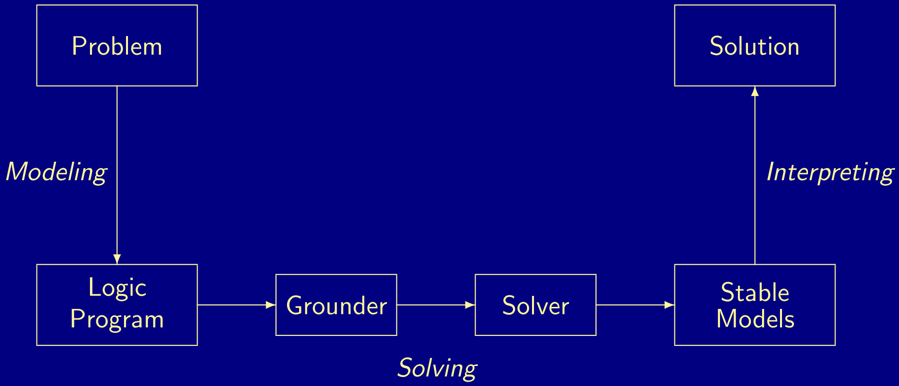
ASP solving


Problem
Solution
Modeling
Interpreting
Logic Program
Grounder
Solver
Stable Models
Solving

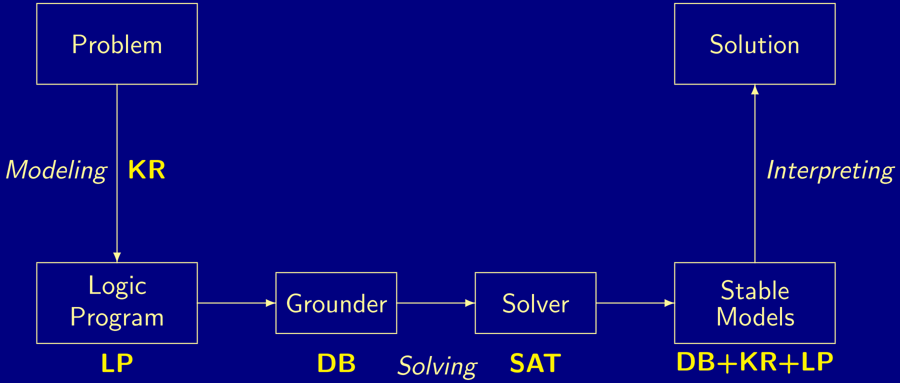
SAT solving


Problem
Solution
Programming
Interpreting
Formula (CNF)
Solver
Classical Models
Solving

Rooting ASP solving



Rooting ASP solving



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Potassco

Two sides of a coin

- ASP as High-level Language
  - Express problem instance(s) as sets of facts
  - Encode problem (class) as a set of rules
  - Read off solutions from stable models of facts and rules
- ASP as Low-level Language
  - Compile a problem into a logic program
  - Solve the original problem by solving its compilation

What is ASP good for?

- Combinatorial search problems in the realm of P, NP, and NP^{NP} (some with substantial amount of data), like
  - Automated Planning
  - Code Optimization
  - Composition of Renaissance Music
  - Database Integration
  - Decision Support for NASA shuttle controllers
  - Model Checking
  - Product Configuration
  - Robotics
  - Systems Biology
  - System Synthesis
  - (industrial) Team-building
  - and many many more

What is ASP good for?

- Combinatorial search problems in the realm of P, NP, and NP^{NP} (some with substantial amount of data), like
  - Automated Planning
  - Code Optimization
  - Composition of Renaissance Music
  - Database Integration
  - Decision Support for NASA shuttle controllers
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What does ASP offer?

- Integration of DB, KR, and SAT techniques
- Succinct, elaboration-tolerant problem representations
  - Rapid application development tool
- Easy handling of dynamic, knowledge intensive applications
  - including: data, frame axioms, exceptions, defaults, closures, etc

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$$\mathbf{ASP = DB{+}LP{+}KR{+}SMT}$$

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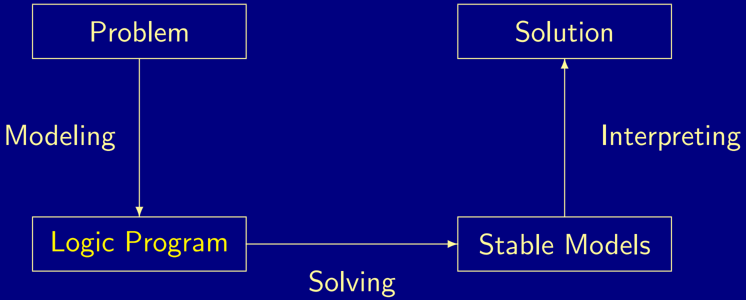
Introduction: Overview

Outline

Problem solving in ASP: Syntax



Normal logic programs

- A logic program, P, over a set $\mathcal{A}$ of atoms is a finite set of rules
- A (normal) rule, r, is of the form

$$a_0 \leftarrow a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n$$

where $0 \leq m \leq n$ and each $a_i \in \mathcal{A}$ is an atom for $0 \leq i \leq n$

$$\begin{aligned} head(r) &= a_0 \\ body(r) &= \{a_1, \ldots, a_m, \sim a_{m+1}, \ldots, \sim a_n\} \\ body(r)^+ &= \{a_1, \ldots, a_m\} \\ body(r)^- &= \{a_{m+1}, \ldots, a_n\} \\ atom(P) &= \bigcup_{r \in P} \left(\{head(r)\} \cup body(r)^+ \cup body(r)^-\right) \\ body(P) &= \{body(r) \mid r \in P\} \end{aligned}$$

A program P is positive if $body(r)^- = \emptyset$ for all $r \in P$

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- Notation

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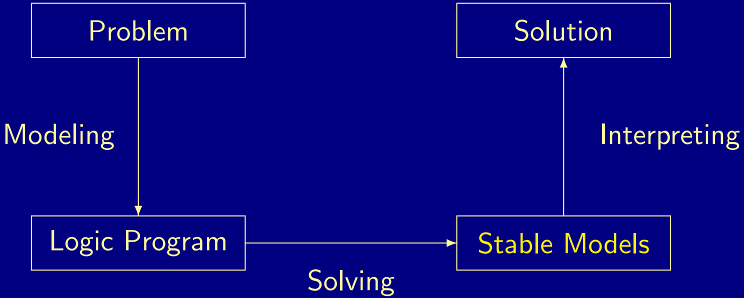
Rough notational convention

We sometimes use the following notation interchangeably
in order to stress the respective view:

	true, false	if	and	or	iff	default negation	classical negation
source code		`:-`	`,`	`|`		`not`	`-`
logic program		$\leftarrow$	$,$	$;$		$\sim$	$\neg$
formula	$\bot, \top$	$\rightarrow$	$\wedge$	$\vee$	$\leftrightarrow$	$\sim$	$\neg$

Outline

Problem solving in ASP: Semantics



Formal Definition

Stable models of positive programs

- A set of atoms X is closed under a positive program P iff for any $r \in P$, $head(r) \in X$ whenever $body(r)^+ \subseteq X$
  - X corresponds to a model of P (seen as a formula)
- The smallest set of atoms which is closed under a positive program P is denoted by $Cn(P)$
  - $Cn(P)$ corresponds to the $\subseteq$-smallest model of P (ditto)
- The set $Cn(P)$ of atoms is the stable model of a *positive* program P

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Some "logical" remarks

- Positive rules are also referred to as definite clauses
  - Definite clauses are disjunctions with exactly one positive atom:

    $$a_0 \vee \neg a_1 \vee \cdots \vee \neg a_m$$

  - A set of definite clauses has a (unique) smallest model
- Horn clauses are clauses with at most one positive atom
  - Every definite clause is a Horn clause but not vice versa
  - Non-definite Horn clauses can be regarded as integrity constraints
  - A set of Horn clauses has a smallest model or none
- This smallest model is the intended semantics of such sets of clauses
  - Given a positive program P, $Cn(P)$ corresponds to the smallest model of the set of definite clauses corresponding to P

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Basic idea

Consider the logical formula Φ and its three (classical) models:

$$\{p, q\}, \{q, r\}, \text{ and } \{p, q, r\}$$

Φ $\boxed{q \;\wedge\; (q \wedge \neg r \rightarrow p)}$

Formula Φ has one stable model, often called answer set:

$$\{p, q\}$$

P_Φ $\boxed{\begin{array}{lll} q & \leftarrow & \\ p & \leftarrow & q, \sim r \end{array}}$

Informally, a set X of atoms is a stable model of a logic program P

- if X is a (classical) model of P and
- if all atoms in X are justified by some rule in P

(rooted in intuitionistic logics HT (Heyting, 1930) and G3 (Gödel, 1932))

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$$P^X = \{head(r) \leftarrow body(r)^+ \mid r \in P \text{ and } body(r)^- \cap X = \emptyset\}$$

- A set X of atoms is a stable model of a program P, if $Cn(P^X) = X$

- Note $Cn(P^X)$ is the $\subseteq$–smallest (classical) model of P^X
- Note Every atom in X is justified by an *"applying rule from P"*

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A closer look at P^X

- In other words, given a set X of atoms from P,

  P^X is obtained from P by deleting

  1. each rule having $\sim a$ in its body with $a \in X$
     and then
  2. all negative atoms of the form $\sim a$
     in the bodies of the remaining rules

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Outline

A first example

$P = \{p \leftarrow p,\ q \leftarrow \sim p\}$

X	P^X	$Cn(P^X)$
$\{\ \}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p\}$	$p \leftarrow p$	$\emptyset$
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$
$\{p, q\}$	$p \leftarrow p$	$\emptyset$

A first example

$P = \{p \leftarrow p,\ q \leftarrow \sim p\}$

X	P^X	$Cn(P^X)$	
$\{\ \}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✗
$\{p\}$	$p \leftarrow p$	$\emptyset$	
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✔
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$P = \{p \leftarrow p,\ q \leftarrow \neg p\}$

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$\{p\}$	$p \leftarrow p$	$\emptyset$	✔
$\{q\}$	$p \leftarrow p$ $q \leftarrow$	$\{q\}$	✔
$\{p, q\}$	$p \leftarrow p$	$\emptyset$	✔

A second example

$$P = \{p \leftarrow \sim q,\ q \leftarrow \sim p\}$$

X	P^X	$Cn(P^X)$
$\{\ \}$	$p \leftarrow$ $q \leftarrow$	$\{p, q\}$
$\{p\}$	$p \leftarrow$	$\{p\}$
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X	P^X	$Cn(P^X)$	
$\{\ \}$	$p \leftarrow$ $q \leftarrow$	$\{p, q\}$	✗
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$\{p\}$	$p \leftarrow$	$\{p\}$	✔
$\{q\}$	$q \leftarrow$	$\{q\}$	✔
$\{p, q\}$		$\emptyset$	✔

A third example

$P = \{p \leftarrow \sim p\}$

X	P^X	$Cn(P^X)$
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Some properties

- A logic program may have zero, one, or multiple stable models!
- If X is a stable model of a logic program P, then X is a model of P (seen as a formula)
- If X and Y are stable models of a *normal* program P, then $X \not\subset Y$

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Outline

Programs with Variables

Let P be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$

- Ground Instances of $r \in P$: Set of variable-free rules obtained by replacing all variables in r by elements from $\mathcal{T}$:

$$ground(r) = \{r\theta \mid \theta : var(r) \to \mathcal{T} \text{ and } var(r\theta) = \emptyset\}$$

  where $var(r)$ stands for the set of all variables occurring in r; θ is a (ground) substitution

- Ground Instantiation of P: $ground(P) = \bigcup_{r \in P} ground(r)$

Programs with Variables

Let P be a logic program

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- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$ (also called alphabet or Herbrand base)
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Let P be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$

- Ground Instances of $r \in P$: Set of variable-free rules obtained by replacing all variables in r by elements from $\mathcal{T}$:

$$ground(r) = \{r\theta \mid \theta : var(r) \rightarrow \mathcal{T} \text{ and } var(r\theta) = \emptyset\}$$

  where $var(r)$ stands for the set of all variables occurring in r;
  θ is a (ground) substitution

- Ground Instantiation of P: $ground(P) = \bigcup_{r \in P} ground(r)$

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An example

$$P = \{\ r(a,b) \leftarrow,\ \ r(b,c) \leftarrow,\ \ t(X,Y) \leftarrow r(X,Y)\ \}$$

$$\mathcal{T} = \{a,b,c\}$$

$$\mathcal{A} = \left\{ \begin{array}{l} r(a,a), r(a,b), r(a,c), r(b,a), r(b,b), r(b,c), r(c,a), r(c,b), r(c,c), \\ t(a,a), t(a,b), t(a,c), t(b,a), t(b,b), t(b,c), t(c,a), t(c,b), t(c,c) \end{array} \right\}$$

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Intelligent Grounding aims at reducing the ground instantiation

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Let P be a normal logic program with variables

- A set X of (ground) atoms is a stable model of P,
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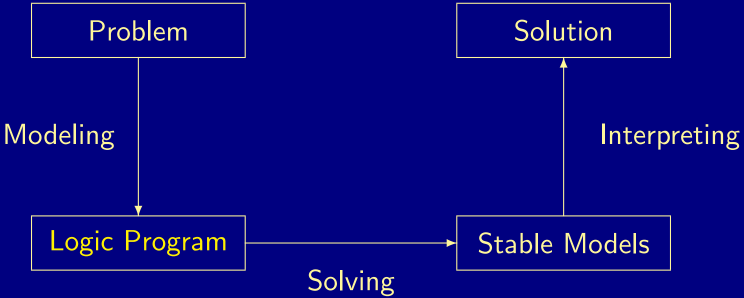
Outline

Problem solving in ASP: Extended Syntax



Language Constructs

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  - `p(X) :- q(X)` over constants $\{a, b, c\}$ stands for
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  - `p :- q(X) : r(X)` given `r(a), r(b), r(c)` stands for
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Potassco

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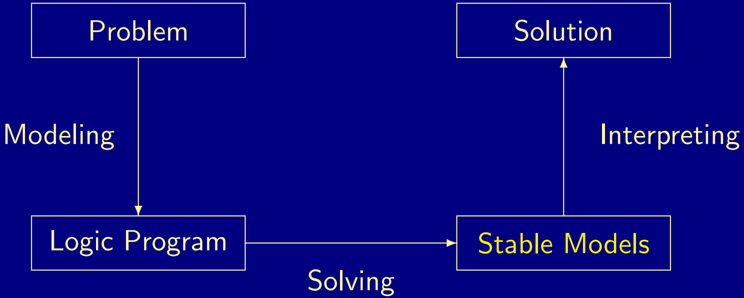
Outline

Problem solving in ASP: Reasoning Modes



Reasoning Modes

- Satisfiability
- Enumeration[†]
- Projection[†]
- Intersection[‡]
- Union[‡]
- Optimization

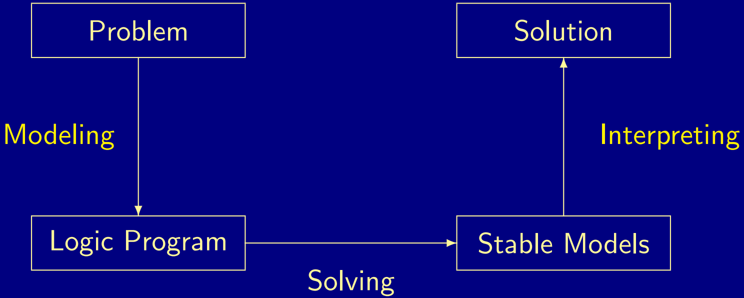
- and combinations of them

[†] without solution recording

[‡] without solution enumeration

Basic Modeling: Overview

Modeling and Interpreting



Modeling

- For solving a problem class **C** for a problem instance **I**, encode
  1. the problem instance **I** as a set $P_{\mathbf{I}}$ of facts and
  2. the problem class **C** as a set $P_{\mathbf{C}}$ of rules

  such that the solutions to **C** for **I** can be (polynomially) extracted from the stable models of $P_{\mathbf{I}} \cup P_{\mathbf{C}}$

- $P_{\mathbf{I}}$ is (still) called problem instance
- $P_{\mathbf{C}}$ is often called the problem encoding

- An encoding $P_{\mathbf{C}}$ is uniform, if it can be used to solve all its problem instances
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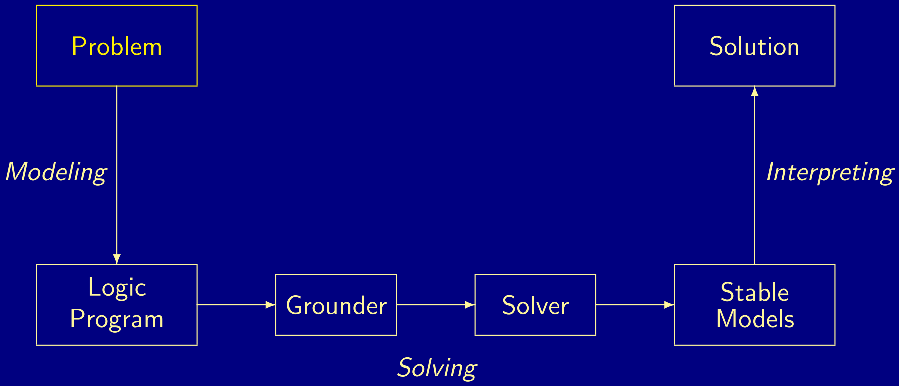
Attention!

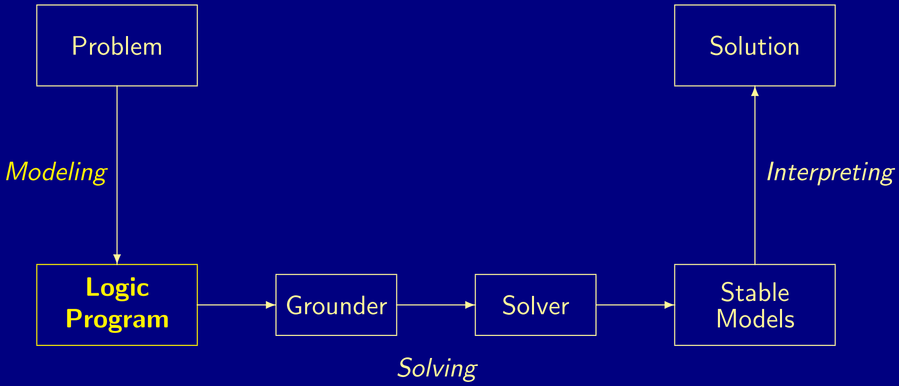
All following examples are written in the language of gringo 3 !

Outline

ASP solving process



ASP solving process



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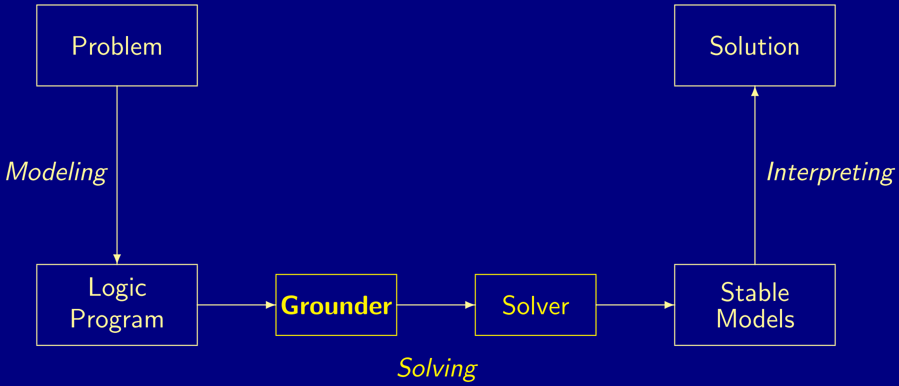

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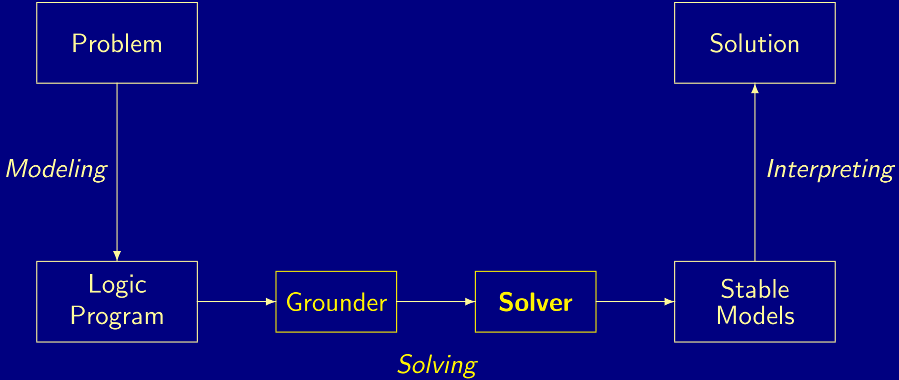

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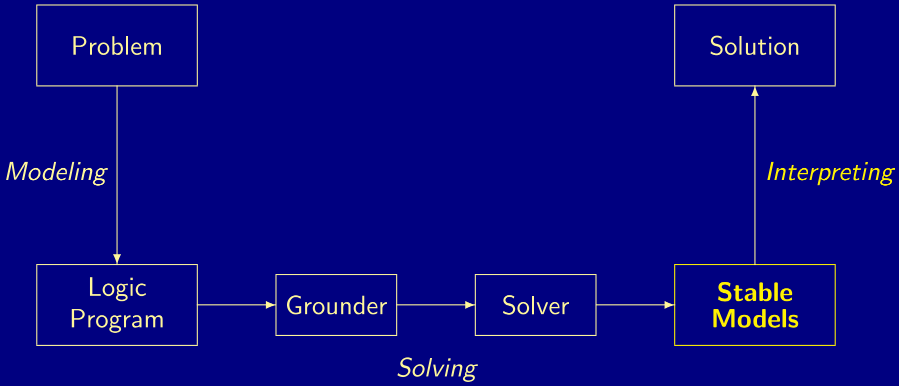

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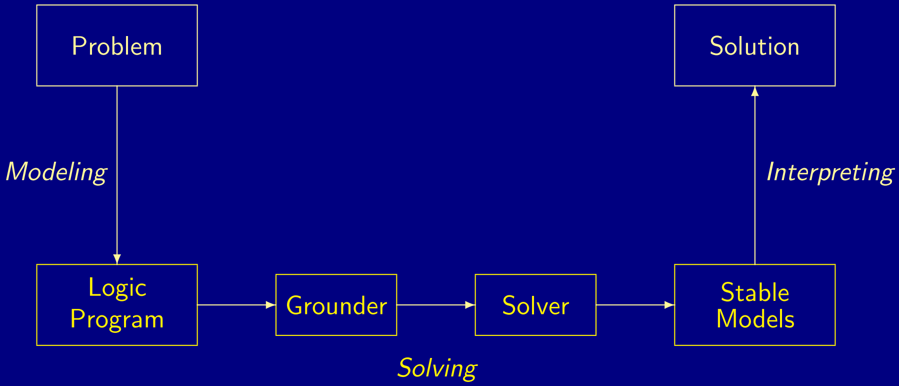

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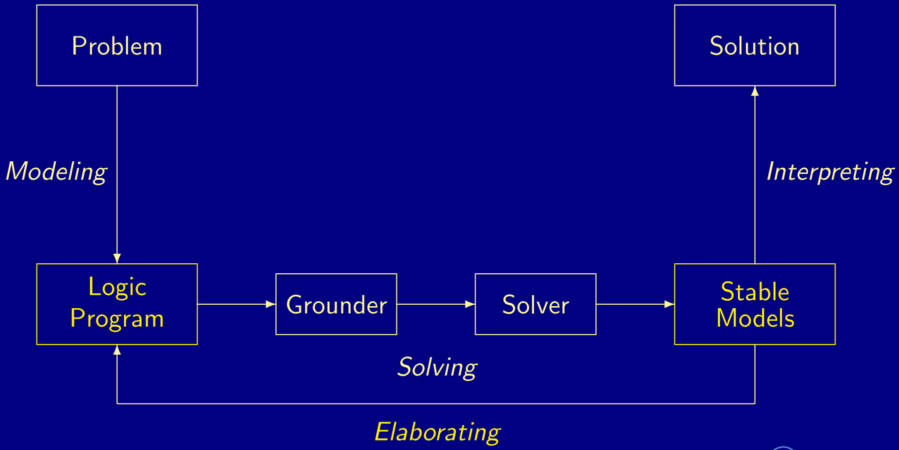
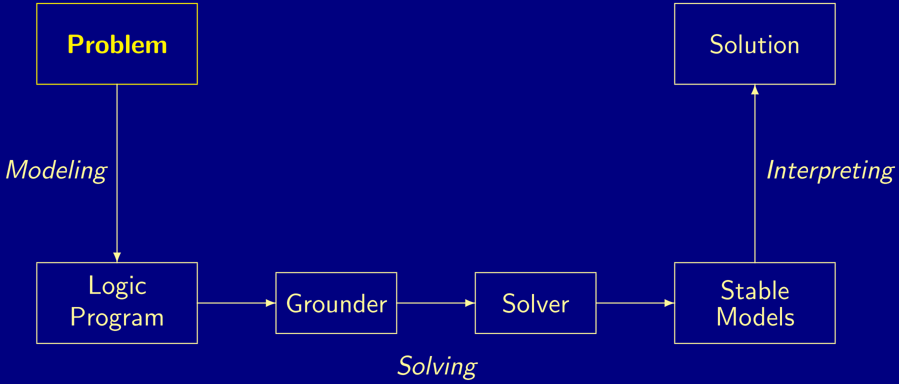


A case-study: Graph coloring



Graph coloring

- Problem instance A graph consisting of nodes and edges

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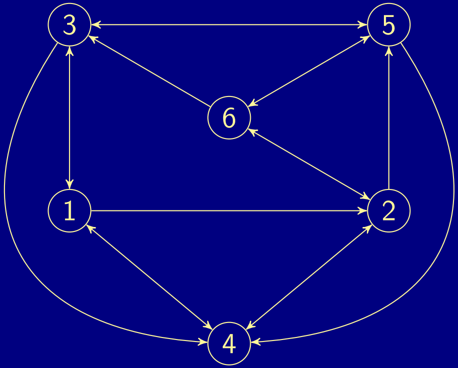

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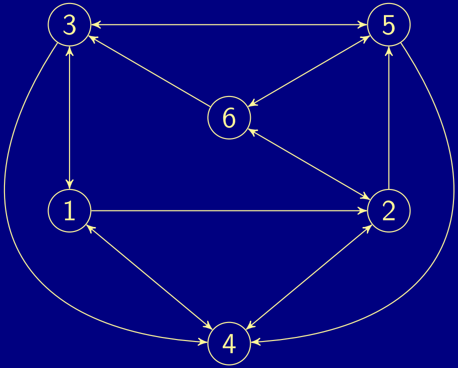

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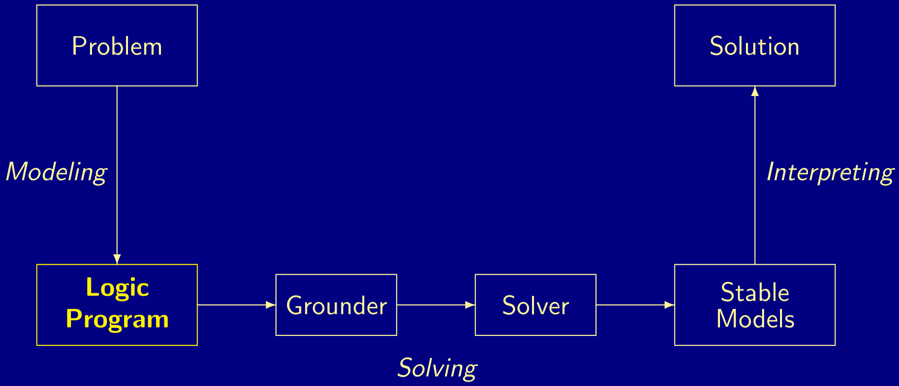
- Problem class Assign each node one color such that no two nodes connected by an edge have the same color

  In other words,

  1. Each node has a unique color
  2. Two connected nodes must not have the same color

ASP solving process



Graph coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
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col(r).   col(b).   col(g).
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Problem instance

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1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
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Problem encoding

Potassco

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edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).
```

Problem instance

```
1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

Problem encoding

Graph coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).
```

Problem instance

```
1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

Problem encoding

color.lp

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).
```

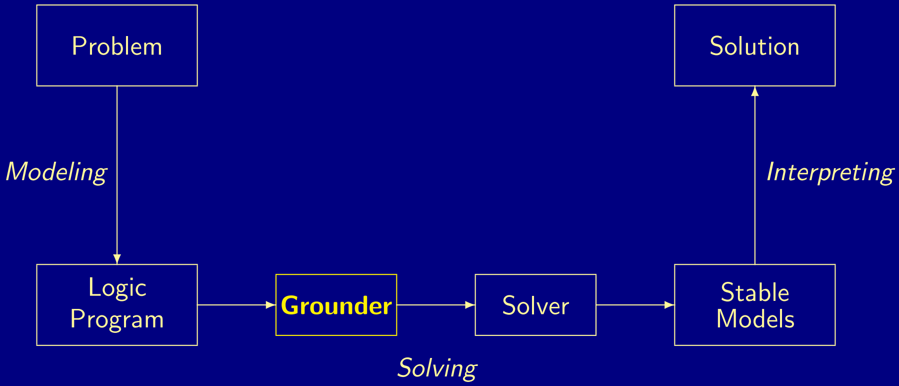
$\left.\right\}$ Problem instance

```
1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

$\left.\right\}$ Problem encoding

ASP solving process



Graph coloring: Grounding

```
$ gringo --text color.lp
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(1,3).  edge(1,4).  edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).  edge(4,1).  edge(4,2).  edge(5,3).
edge(5,4).  edge(5,6).  edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

 :- color(1,r), color(2,r).  :- color(2,g), color(5,g).  ...  :- color(6,r), color(2,r).
 :- color(1,b), color(2,b).  :- color(2,r), color(6,r).       :- color(6,b), color(2,b).
 :- color(1,g), color(2,g).  :- color(2,b), color(6,b).       :- color(6,g), color(2,g).
 :- color(1,r), color(3,r).  :- color(2,g), color(6,g).       :- color(6,r), color(3,r).
 :- color(1,b), color(3,b).  :- color(3,r), color(1,r).       :- color(6,b), color(3,b).
 :- color(1,g), color(3,g).  :- color(3,b), color(1,b).       :- color(6,g), color(3,g).
 :- color(1,r), color(4,r).  :- color(3,g), color(1,g).       :- color(6,r), color(5,r).
 :- color(1,b), color(4,b).  :- color(3,r), color(4,r).       :- color(6,b), color(5,b).
 :- color(1,g), color(4,g).  :- color(3,b), color(4,b).       :- color(6,g), color(5,g).
 :- color(2,r), color(4,r).  :- color(3,g), color(4,g).
 :- color(2,b), color(4,b).  :- color(3,r), color(5,r).
 :- color(2,g), color(4,g).  :- color(3,b), color(5,b).
```

Graph coloring: Grounding

```
$ gringo --text color.lp
```

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

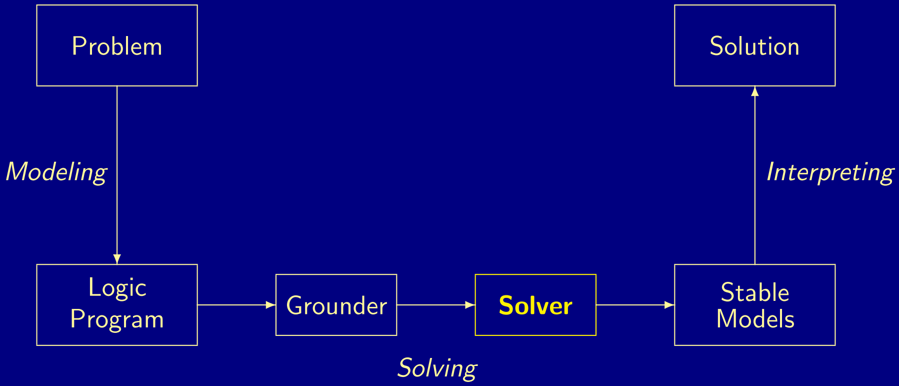
edge(1,2).  edge(1,3).  edge(1,4).  edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).  edge(4,1).  edge(4,2).  edge(5,3).
edge(5,4).  edge(5,6).  edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

 :- color(1,r), color(2,r).  :- color(2,g), color(5,g).  ...  :- color(6,r), color(2,r).
 :- color(1,b), color(2,b).  :- color(2,r), color(6,r).       :- color(6,b), color(2,b).
 :- color(1,g), color(2,g).  :- color(2,b), color(6,b).       :- color(6,g), color(2,g).
 :- color(1,r), color(3,r).  :- color(2,g), color(6,g).       :- color(6,r), color(3,r).
 :- color(1,b), color(3,b).  :- color(3,r), color(1,r).       :- color(6,b), color(3,b).
 :- color(1,g), color(3,g).  :- color(3,b), color(1,b).       :- color(6,g), color(3,g).
 :- color(1,r), color(4,r).  :- color(3,g), color(1,g).       :- color(6,r), color(5,r).
 :- color(1,b), color(4,b).  :- color(3,r), color(4,r).       :- color(6,b), color(5,b).
 :- color(1,g), color(4,g).  :- color(3,b), color(4,b).       :- color(6,g), color(5,g).
 :- color(2,r), color(4,r).  :- color(3,g), color(4,g).
 :- color(2,b), color(4,b).  :- color(3,r), color(5,r).
 :- color(2,g), color(4,g).  :- color(3,b), color(5,b).
```

ASP solving process



Graph coloring: Solving

```
$ gringo color.lp | clasp 0
```

```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,g) color(4,b) color(3,r) color(2,r) color(1,g)
Answer: 2
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,g) color(4,r) color(3,b) color(2,b) color(1,g)
Answer: 3
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,b) color(4,g) color(3,r) color(2,r) color(1,b)
Answer: 4
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,b) color(4,r) color(3,g) color(2,g) color(1,b)
Answer: 5
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,r) color(4,g) color(3,b) color(2,b) color(1,r)
Answer: 6
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
SATISFIABLE

Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
```

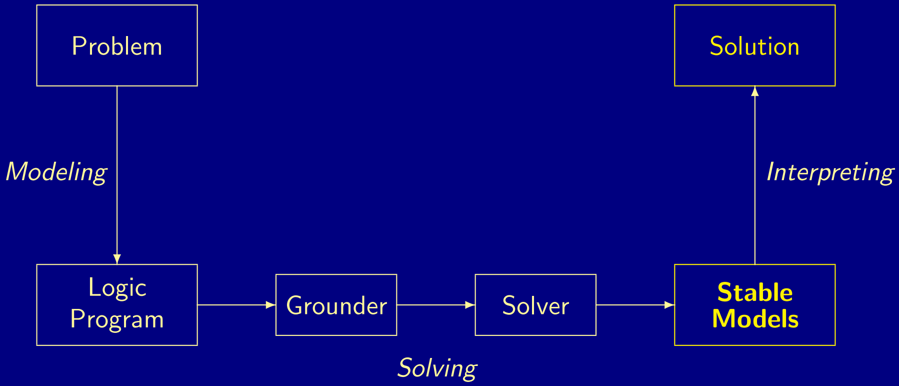
Graph coloring: Solving

```
$ gringo color.lp | clasp 0
```

```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,g) color(4,b) color(3,r) color(2,r) color(1,g)
Answer: 2
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,g) color(4,r) color(3,b) color(2,b) color(1,g)
Answer: 3
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,b) color(4,g) color(3,r) color(2,r) color(1,b)
Answer: 4
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,b) color(4,r) color(3,g) color(2,g) color(1,b)
Answer: 5
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,r) color(4,g) color(3,b) color(2,b) color(1,r)
Answer: 6
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
SATISFIABLE

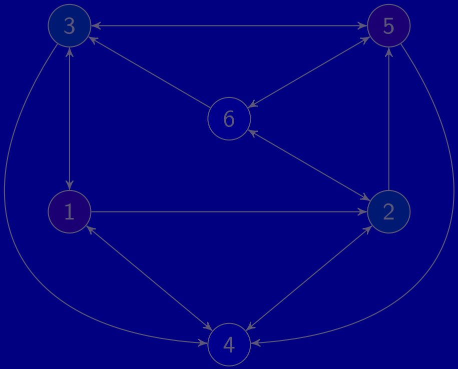
Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
```

ASP solving process



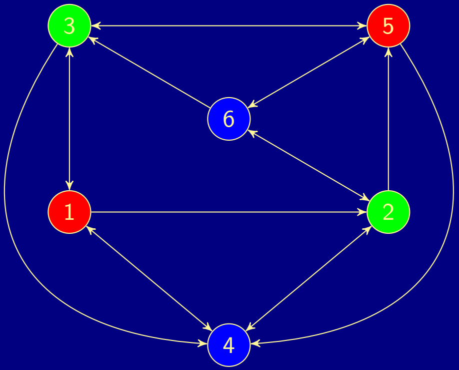
A coloring

```
Answer: 6
edge(1,2) ... col(r) ... node(1) ...                                   \
color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
```



A coloring

```
Answer: 6
edge(1,2) ... col(r) ... node(1) ...                                    \
color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
```



Outline

Basic methodology

Methodology

Generate and Test (or: Guess and Check)

Generator
: Generate potential stable model candidates
(typically through non-deterministic constructs)

Tester
: Eliminate invalid candidates
(typically through integrity constraints)

Nutshell

Logic program = Data + Generator + Tester (+ Optimizer)

Basic methodology

Methodology

Generate and Test (or: Guess and Check)

- Generator: Generate potential stable model candidates (typically through non-deterministic constructs)
- Tester: Eliminate invalid candidates (typically through integrity constraints)

Nutshell

Logic program = Data + Generator + Tester (+ Optimizer)

Outline

Satisfiability testing

- Problem Instance: A propositional formula ϕ in CNF
- Problem Class: Is there an assignment of propositional variables to true and false such that a given formula ϕ is true

- Example: Consider formula

$$(a \vee \neg b) \wedge (\neg a \vee b)$$

- Logic Program:

Generator	**Tester**	**Stable models**
$\{a, b\} \leftarrow$	$\leftarrow \sim a, b$	$X_1 = \{a, b\}$
	$\leftarrow a, \sim b$	$X_2 = \{\}$

Satisfiability testing

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Generator	**Tester**	**Stable models**
$\{ a, b \} \leftarrow$	$\leftarrow \; {\sim}a, b$	$X_1 = \{a, b\}$
	$\leftarrow \; a, {\sim}b$	$X_2 = \{\}$

Potassco

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- Example: Consider formula

$$(a \vee \neg b) \wedge (\neg a \vee b)$$

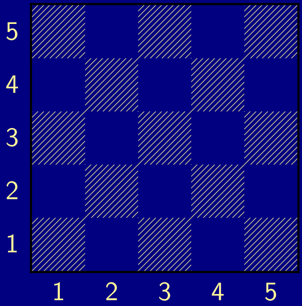
- Logic Program:

Generator	**Tester**	**Stable models**
$\{ a, b \} \leftarrow$	$\leftarrow \; \sim a, b$	$X_1 = \{a, b\}$
	$\leftarrow \; a, \sim b$	$X_2 = \{\}$

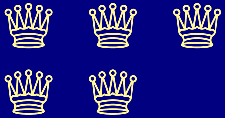
Potassco

Outline

The n-Queens Problem



- Place n queens on an $n \times n$ chess board
- Queens must not attack one another



Defining the Field

queens.lp

```
row(1..n).
col(1..n).
```

- Create file `queens.lp`
- Define the field
  - n rows
  - n columns

Defining the Field

Running . . .

```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
SATISFIABLE

Models      : 1
Time        : 0.000
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.000
```

Placing some Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
```

- Guess a solution candidate
  by placing some queens on the board

Placing some Queens

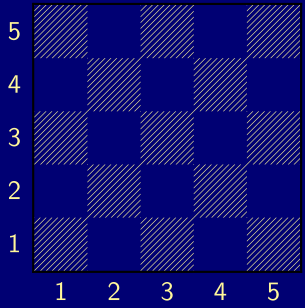
Running . . .

```
$ gringo queens.lp --const n=5 | clasp 3
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(1,1)
Answer: 3
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(2,1)
SATISFIABLE

Models     : 3+
...
```

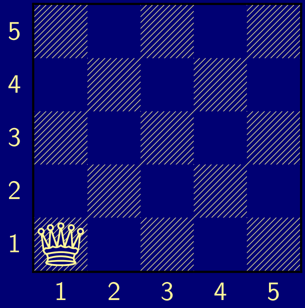
Placing some Queens: Answer 1

Answer 1



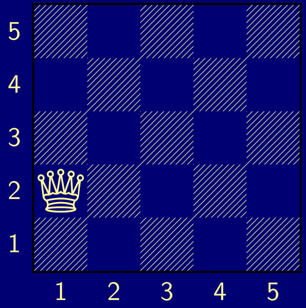
Placing some Queens: Answer 2

Answer 2



Placing some Queens: Answer 3

Answer 3



Placing n Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
```

- Place exactly n queens on the board

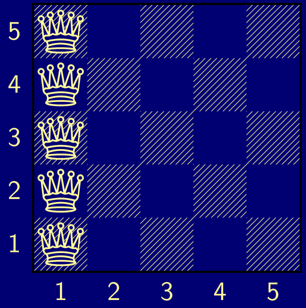
Placing n Queens

Running ...

```
$ gringo queens.lp --const n=5 | clasp 2
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,1) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(1,2) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
...
```

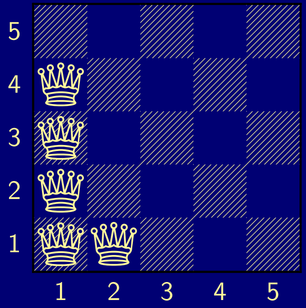
Placing *n* Queens: Answer 1

Answer 1



Placing n Queens: Answer 2

Answer 2



Horizontal and Vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
```

- Forbid horizontal attacks
- Forbid vertical attacks

Horizontal and Vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
```

- Forbid horizontal attacks
- Forbid vertical attacks

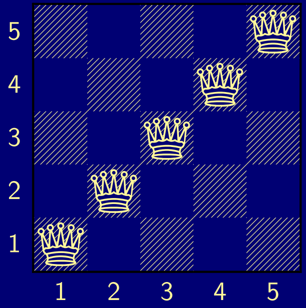
Horizontal and Vertical Attack

Running ...

```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,5) queen(4,4) queen(3,3) \
queen(2,2) queen(1,1)
...
```

Horizontal and Vertical Attack: Answer 1

Answer 1



Diagonal Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
:- queen(I,J), queen(II,JJ), (I,J) != (II,JJ), I-J == II-JJ.
:- queen(I,J), queen(II,JJ), (I,J) != (II,JJ), I+J == II+JJ.
```

- Forbid diagonal attacks

Diagonal Attack

Running ...

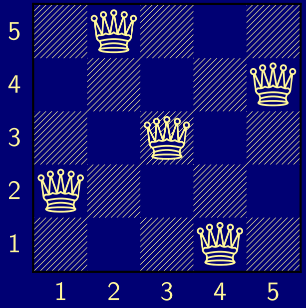
```
$ gringo queens.lp --const n=5 | clasp
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(4,5) queen(1,4) queen(3,3) queen(5,2) queen(2,1)
SATISFIABLE

Models      : 1+
Time        : 0.000
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.000
```

Potassco

Diagonal Attack: Answer 1

Answer 1



Optimizing

queens-opt.lp

```
1 { queen(I,1..n) } 1 :- I = 1..n.
1 { queen(1..n,J) } 1 :- J = 1..n.
 :- 2 { queen(D-J,J) }, D = 2..2*n.
 :- 2 { queen(D+J,J) }, D = 1-n..n-1.
```

- Encoding can be optimized
- Much faster to solve

And sometimes it rocks

```
$ clingo -c n=5000 queens-opt-diag.lp --config=jumpy -q --stats=3
clingo version 4.1.0
Solving...
SATISFIABLE

Models       : 1+
Time         : 3758.143s (Solving: 1905.22s 1st Model: 1896.20s Unsat: 0.00s)
CPU Time     : 3758.320s

Choices      : 288594554
Conflicts    : 3442   (Analyzed: 3442)
Restarts     : 17     (Average: 202.47 Last: 3442)
Model-Level  : 7594728.0
Problems     : 1      (Average Length: 0.00 Splits: 0)
Lemmas       : 3442   (Deleted: 0)
  Binary     : 0      (Ratio:   0.00%)
  Ternary    : 0      (Ratio:   0.00%)
  Conflict   : 3442   (Average Length: 229056.5 Ratio: 100.00%)
  Loop       : 0      (Average Length:    0.0 Ratio:   0.00%)
  Other      : 0      (Average Length:    0.0 Ratio:   0.00%)

Atoms        : 75084857 (Original: 75069989 Auxiliary: 14868)
Rules        : 100129956 (1: 50059992/100090100 2: 39990/29856 3: 10000/10000)
Bodies       : 25090103
Equivalences : 125029999 (Atom=Atom: 50009999 Body=Body: 0 Other: 75020000)
Tight        : Yes
Variables    : 25024868 (Eliminated: 11781 Frozen: 25000000)
Constraints  : 66664  (Binary: 35.6% Ternary:  0.0% Other: 64.4%)

Backjumps    : 3442   (Average: 681.19 Max: 169512 Sum: 2344658)
  Executed   : 3442   (Average: 681.19 Max: 169512 Sum: 2344658 Ratio: 100.00%)
```

Outline

Traveling Salesperson

```
node(1..6).

edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
edge(4,1;2).    edge(5,3;4;6).  edge(6,2;3;5).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).
```

Potassco

Traveling Salesperson

```
node(1..6).

edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
edge(4,1;2).    edge(5,3;4;6).  edge(6,2;3;5).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
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Traveling Salesperson

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edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
edge(4,1;2).    edge(5,3;4;6).  edge(6,2;3;5).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
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```

Traveling Salesperson

```
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(X).
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(Y).

reached(Y) :- cycle(1,Y).
reached(Y) :- cycle(X,Y), reached(X).

:- node(Y), not reached(Y).

#minimize [ cycle(X,Y) = C : cost(X,Y,C) ].
```

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Outline

13 ASP solving process

14 Methodology
- Satisfiability
- Queens
- Traveling Salesperson
- Reviewer Assignment
- Planning

Reviewer Assignment
by Ilkka Niemelä

```
reviewer(r1). paper(p1). classA(r1,p1). classB(r1,p2). coi(r1,p3).
reviewer(r2). paper(p2). classA(r1,p3). classB(r1,p4). coi(r1,p6).
...

3 { assigned(P,R) : reviewer(R) } 3 :-  paper(P).

 :- assigned(P,R), coi(R,P).
 :- assigned(P,R), not classA(R,P), not classB(R,P).
 :- not 6 { assigned(P,R) : paper(P) } 9, reviewer(R).

assignedB(P,R) :-  classB(R,P), assigned(P,R).
 :- 3 { assignedB(P,R) : paper(P) }, reviewer(R).

#minimize { assignedB(P,R) : paper(P) : reviewer(R) }.
```

Potassco

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Potassco

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Outline

Simplistic STRIPS Planning

```
time(1..k).    lasttime(T) :- time(T), not time(T+1).

fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).    query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

1 { occ(A,T) : action(A) } 1 :- time(T).
 :- occ(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- holds(F,T-1), not nolds(F,T), time(T).
holds(F,T) :- occ(A,T), add(A,F).
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Language: Overview

Outline

Basic language extensions

- The expressiveness of a language can be enhanced by introducing new constructs
- To this end, we must address the following issues:
  - What is the syntax of the new language construct?
  - What is the semantics of the new language construct?
  - How to implement the new language construct?
- A way of providing semantics is to furnish a translation removing the new constructs, eg. classical negation
- This translation might also be used for implementing the language extension

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Outline

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Integrity constraint

- Idea Eliminate unwanted solution candidates
- Syntax An integrity constraint is of the form

$$\leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n$$

where $0 \leq m \leq n$ and each a_i is an atom for $1 \leq i \leq n$

- Example `:- edge(3,7), color(3,red), color(7,red).`
- Embedding The above integrity constraint can be turned into the normal rule

$$x \leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n, \sim x$$

where x is a new symbol, that is, $x \notin \mathcal{A}$.

- Another example $P = \{a \leftarrow \sim b,\ b \leftarrow \sim a\}$
  versus $P' = P \cup \{\leftarrow a\}$ and $P'' = P \cup \{\leftarrow \sim a\}$

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Choice rule

- Idea Choices over subsets
- Syntax A choice rule is of the form

$$\{a_1, \dots, a_m\} \leftarrow a_{m+1}, \dots, a_n, \sim a_{n+1}, \dots, \sim a_o$$

where $0 \leq m \leq n \leq o$ and each a_i is an atom for $1 \leq i \leq o$

- Informal meaning If the body is satisfied by the stable model at hand, then any subset of $\{a_1, \dots, a_m\}$ can be included in the stable model
- Example `{ buy(pizza), buy(wine), buy(corn) } :- at(grocery).`
- Another Example $P = \{\{a\} \leftarrow b,\ b \leftarrow\}$ has two stable models: $\{b\}$ and $\{a, b\}$

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Embedding in normal rules

- A choice rule of form

$$\{a_1, \dots, a_m\} \leftarrow a_{m+1}, \dots, a_n, \sim a_{n+1}, \dots, \sim a_o$$

can be translated into $2m + 1$ normal rules

$$\begin{array}{rcl} a' & \leftarrow & a_{m+1}, \dots, a_n, \sim a_{n+1}, \dots, \sim a_o \end{array}$$

$$\begin{array}{rclcrcl} a_1 & \leftarrow & a', \sim\overline{a_1} & \dots & a_m & \leftarrow & a', \sim\overline{a_m} \\ \overline{a_1} & \leftarrow & \sim a_1 & \dots & \overline{a_m} & \leftarrow & \sim a_m \end{array}$$

by introducing new atoms $a', \overline{a_1}, \dots, \overline{a_m}$.

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Outline

Cardinality rule

- Idea Control (lower) cardinality of subsets
- Syntax A cardinality rule is the form

$$a_0 \leftarrow l \; \{ \; a_1, \ldots, a_m, {\sim}a_{m+1}, \ldots, {\sim}a_n \; \}$$

  where $0 \leq m \leq n$ and each a_i is an atom for $1 \leq i \leq n$;
  l is a non-negative integer.

- Informal meaning The head atom belongs to the stable model, if at least l elements of the body are included in the stable model
- Note l acts as a lower bound on the body
- Example `pass(c42) :- 2 { pass(a1), pass(a2), pass(a3) }.`
- Another Example $P = \{a \leftarrow 1\{b, c\}, \; b \leftarrow\}$ has stable model $\{a, b\}$

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Embedding in normal rules

- Replace each cardinality rule

$$a_0 \leftarrow l \; \{ \; a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n \; \}$$

by $\quad a_0 \leftarrow ctr(1, l)$

where atom $ctr(i, j)$ represents the fact that at least j of the literals having an equal or greater index than i, are in a stable model

- The definition of $ctr/2$ is given for $0 \leq k \leq l$ by the rules

$$\begin{array}{rcll} ctr(i, k{+}1) & \leftarrow & ctr(i+1, k), a_i & \\ ctr(i, k) & \leftarrow & ctr(i+1, k) & \text{for } 1 \leq i \leq m \\ ctr(j, k{+}1) & \leftarrow & ctr(j+1, k), \sim a_j & \\ ctr(j, k) & \leftarrow & ctr(j+1, k) & \text{for } m+1 \leq j \leq n \\ ctr(n+1, 0) & \leftarrow & & \end{array}$$

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where atom $ctr(i, j)$ represents the fact that at least j of the literals having an equal or greater index than i, are in a stable model

- The definition of $ctr/2$ is given for $0 \leq k \leq l$ by the rules

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Embedding in normal rules

- Replace each cardinality rule

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An example

- Program $\{a \leftarrow,\ c \leftarrow 1\ \{a, b\}\}$ has the stable model $\{a, c\}$
- Translating the cardinality rule yields the rules

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having stable model $\{a, ctr(3,0), ctr(2,0), ctr(1,0), ctr(1,1), c\}$

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... and vice versa

- A normal rule

$$a_0 \leftarrow a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n,$$

can be represented by the cardinality rule

$$a_0 \leftarrow n \; \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\}$$

Cardinality rules with upper bounds

- A rule of the form

$$a_0 \leftarrow l \; \{ \; a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n \; \} \; u$$

where $0 \le m \le n$ and each a_i is an atom for $1 \le i \le n$;
l and u are non-negative integers

stands for

$$\begin{array}{rcl} a_0 & \leftarrow & b, \sim c \\ b & \leftarrow & l \; \{ \; a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n \; \} \\ c & \leftarrow & u{+}1 \; \{ \; a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n \; \} \end{array}$$

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- The single constraint in the body of the above cardinality rule is referred to as a cardinality constraint

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  where $0 \leq m \leq n$ and each a_i is an atom for $1 \leq i \leq n$; l and u are non-negative integers

- Informal meaning A cardinality constraint is satisfied by a stable model X, if the number of its contained literals satisfied by X is between l and u (inclusive)

- In other words, if

$$l \leq |\,(\{a_1, \ldots, a_m\} \cap X) \cup (\{a_{m+1}, \ldots, a_n\} \setminus X)\,| \leq u$$

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Cardinality constraints as heads

- A rule of the form

$$l\ \{a_1,\dots,a_m,\sim a_{m+1},\dots,\sim a_n\}\ u \leftarrow a_{n+1},\dots,a_o,\sim a_{o+1},\dots,\sim a_p$$

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- Example `1 { color(v42,red),color(v42,green),color(v42,blue) } 1.`

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Outline

Weight rule

- Syntax A weight rule is the form

$$a_0 \leftarrow l \; \{ \; a_1 = w_1, \ldots, a_m = w_m, {\sim}a_{m+1} = w_{m+1}, \ldots, {\sim}a_n = w_n \; \}$$

  where $0 \leq m \leq n$ and each a_i is an atom;
  l and w_i are integers for $1 \leq i \leq n$

- A weighted literal, $\ell_i = w_i$, associates each literal ℓ_i with a weight w_i

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- Meaning A weight constraint is satisfied by a stable model X, if

$$l \leq \left(\sum_{1 \leq i \leq m, a_i \in X} w_i + \sum_{m < i \leq n, a_i \notin X} w_i \right) \leq u$$

- Note (Cardinality and) weight constraints amount to constraints on (count and) sum aggregate functions

- Example `10 [course(db)=6,course(ai)=6,course(project)=8,course(xml)=3] 20`

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Outline

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Conditional literals (in *lparse* & *gringo* 3)

- Syntax A conditional literal is of the form

  $$\ell : \ell_1 : \cdots : \ell_n$$

  where ℓ and ℓ_i are literals for $0 \leq i \leq n$
- Informal meaning A conditional literal can be regarded as the list of elements in the set $\{\ell \mid \ell_1, \dots, \ell_n\}$
- Note The expansion of conditional literals is context dependent
- Example Given '`p(1). p(2). p(3). q(2).`'

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  r(X):p(X):not q(X) :- r(X):p(X):not q(X), 1 {r(X):p(X):not q(X)}.
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Conditional literals (in *lparse* & *gringo* 3)

- Syntax A conditional literal is of the form

  $$\ell : \ell_1 : \cdots : \ell_n$$

  where ℓ and ℓ_i are literals for $0 \leq i \leq n$

- Informal meaning A conditional literal can be regarded as the list of elements in the set $\{\ell \mid \ell_1, \dots, \ell_n\}$
- Note The expansion of conditional literals is context dependent
- Example Given '`p(1). p(2). p(3). q(2).`'

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Optimization statement

- Idea Express cost functions subject to minimization and/or maximization
- Syntax A minimize statement is of the form

$$minimize\{\ \ell_1 = w_1@p_1, \dots, \ell_n = w_n@p_n\ \}.$$

where each ℓ_i is a literal; and w_i and p_i are integers for $1 \leq i \leq n$

Priority levels, p_i, allow for representing lexicographically ordered minimization objectives

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- Example When configuring a computer, we may want to maximize hard disk capacity, while minimizing price

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#maximize[ hd(1)=250@1, hd(2)=500@1, hd(3)=750@1, hd(4)=1000@1 ].
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The priority levels indicate that (minimizing) price is more important than (maximizing) capacity

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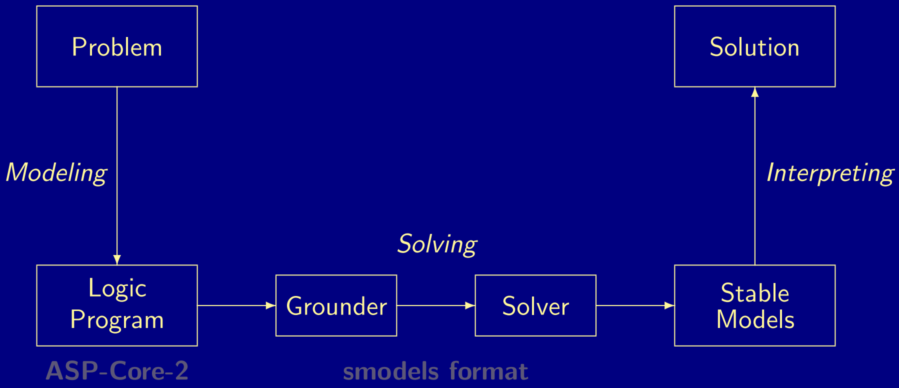
smodels format

- Logic programs in *smodels* format consist of
  - normal rules
  - choice rules
  - cardinality rules
  - weight rules
  - optimization statements
- Such a format is obtained by grounders *lparse* and *gringo*

Outline

ASP-Core-2



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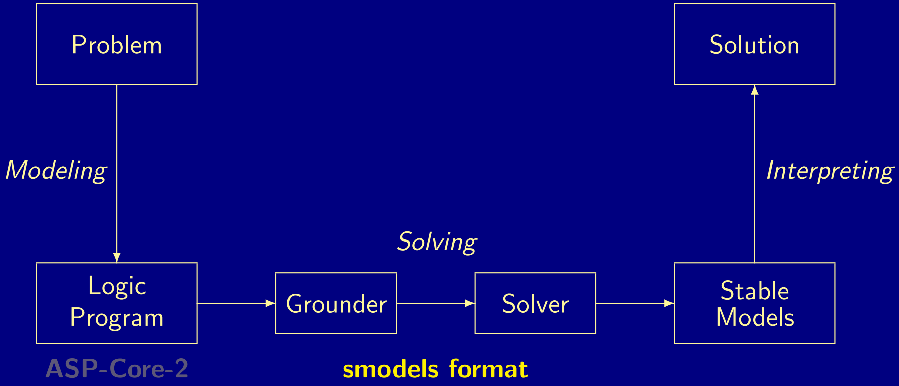

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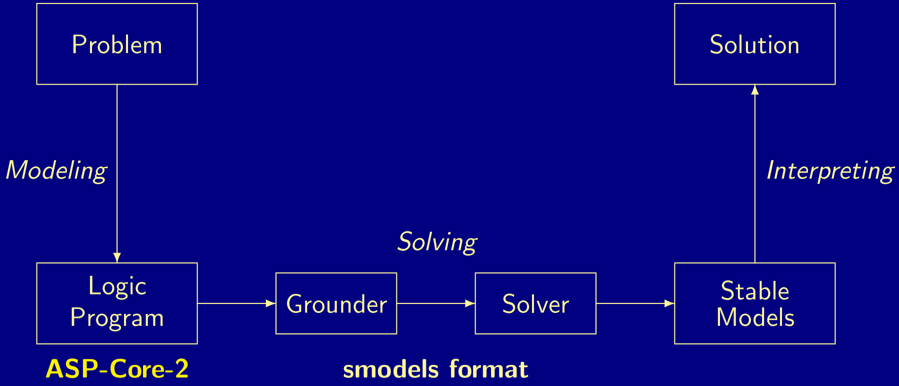

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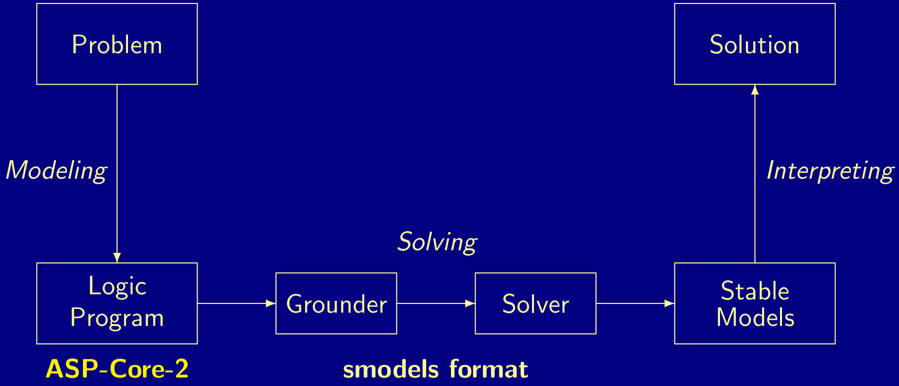

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Aggregates

- Syntax ASP-Core-2 aggregates are of the form

$$t_1 \prec_1 \#\mathtt{A}\{t_{1_1},\dots,t_{m_1} : \ell_{1_1},\dots,\ell_{n_1}\} \prec_2 t_2$$

  where
  - $\#\mathtt{A} \in \{\#\mathtt{count}, \#\mathtt{sum}, \#\mathtt{max}, \#\mathtt{min}\}$
  - $\prec_1, \prec_2 \in \{<, \leq, =, \neq, >, \geq\}$
  - $t_{1_1},\dots,t_{m_1}$ and t_1, t_2 are terms
  - $\ell_{1_1},\dots,\ell_{n_1}$ are literals

- Example Weight constraint

```
10 [course(db)=6,course(ai)=6,course(project)=8,course(xml)=3] 20
```

  is written as an ASP-Core-2 aggregate as

$$10 \leq \#\mathtt{sum}\{\mathtt{6,db:course(db);\ 6,ai:course(ai);}$$
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$$:\sim a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n. \, [w@p, t_1, \dots, t_m]$$

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- Example Minimize statement

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#minimize[ hd(1)=30@2, hd(2)=40@2, hd(3)=60@2, hd(4)=80@2 ].
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gringo 4

- The input language of *gringo* series 4 comprises
  - ASP-Core-2
  - concepts from *gringo* 3 (conditional literals, #show directives, ...)
- Example The *gringo* 3 rule

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r(X):p(X):not q(X) :- r(X):p(X):not q(X), 1 {r(X):p(X):not q(X)}.
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can be written as follows in the language of *gringo* 4:

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Term-based #show directives as in

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The languages of *gringo* 3 and 4 are not fully compatible

Many example programs given in this tutorial are written for *gringo* 3

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Language Extensions: Overview

Outline

Motivation

- Classical versus default negation
  - Symbol $\neg$ and $\sim$
  - Idea
    - $\neg a \quad \approx \quad \neg a \in X$
    - $\sim a \quad \approx \quad a \notin X$
  - Example
    - $cross \leftarrow \neg train$
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Classical negation

- We consider logic programs in negation normal form
  - That is, classical negation is applied to atoms only
- Given an alphabet $\mathcal{A}$ of atoms, let $\overline{\mathcal{A}} = \{\neg a \mid a \in \mathcal{A}\}$ such that $\mathcal{A} \cap \overline{\mathcal{A}} = \emptyset$
- Given a program P over $\mathcal{A}$, classical negation is encoded by adding

$$P^{\neg} = \{a \leftarrow b, \neg b \mid a \in (\mathcal{A} \cup \overline{\mathcal{A}}), b \in \mathcal{A}\}$$

- A set X of atoms is a stable model of a program P over $\mathcal{A} \cup \overline{\mathcal{A}}$, if X is a stable model of $P \cup P^{\neg}$

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An example

- The program

$$P \;=\; \{a \leftarrow {\sim}b,\; b \leftarrow {\sim}a\} \cup \{c \leftarrow b,\; \neg c \leftarrow b\}$$

induces

$$P^{\neg} = \left\{ \begin{array}{rcl rcl rcl} a & \leftarrow & a, \neg a & a & \leftarrow & b, \neg b & a & \leftarrow & c, \neg c \\ \neg a & \leftarrow & a, \neg a & \neg a & \leftarrow & b, \neg b & \neg a & \leftarrow & c, \neg c \\ b & \leftarrow & a, \neg a & b & \leftarrow & b, \neg b & b & \leftarrow & c, \neg c \\ \neg b & \leftarrow & a, \neg a & \neg b & \leftarrow & b, \neg b & \neg b & \leftarrow & c, \neg c \\ c & \leftarrow & a, \neg a & c & \leftarrow & b, \neg b & c & \leftarrow & c, \neg c \\ \neg c & \leftarrow & a, \neg a & \neg c & \leftarrow & b, \neg b & \neg c & \leftarrow & c, \neg c \end{array} \right\}$$

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$$P^{\neg} = \left\{ \begin{array}{rclrclrcl} a & \leftarrow & a, \neg a & a & \leftarrow & b, \neg b & a & \leftarrow & c, \neg c \\ \neg a & \leftarrow & a, \neg a & \neg a & \leftarrow & b, \neg b & \neg a & \leftarrow & c, \neg c \\ b & \leftarrow & a, \neg a & b & \leftarrow & b, \neg b & b & \leftarrow & c, \neg c \\ \neg b & \leftarrow & a, \neg a & \neg b & \leftarrow & b, \neg b & \neg b & \leftarrow & c, \neg c \\ c & \leftarrow & a, \neg a & c & \leftarrow & b, \neg b & c & \leftarrow & c, \neg c \\ \neg c & \leftarrow & a, \neg a & \neg c & \leftarrow & b, \neg b & \neg c & \leftarrow & c, \neg c \end{array} \right\}$$

- The stable models of P are given by the ones of $P \cup P^{\neg}$, viz $\{a\}$

Properties

- The only inconsistent stable "model" is $X = \mathcal{A} \cup \overline{\mathcal{A}}$
- Note Strictly speaking, an inconsistemt set like $\mathcal{A} \cup \overline{\mathcal{A}}$ is not a model
- For a logic program P over $\mathcal{A} \cup \overline{\mathcal{A}}$, exactly one of the following two cases applies:
  1. All stable models of P are consistent or
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Train spotting

- $P_1 = \{cross \leftarrow \sim train\}$
  - stable model: $\{cross\}$
- $P_2 = \{cross \leftarrow \neg train\}$
  - stable model: $\emptyset$
- $P_3 = \{cross \leftarrow \neg train, \ \neg train \leftarrow\}$
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Default negation in rule heads

- We consider logic programs with default negation in rule heads
- Given an alphabet $\mathcal{A}$ of atoms, let $\widetilde{\mathcal{A}} = \{\widetilde{a} \mid a \in \mathcal{A}\}$ such that $\mathcal{A} \cap \widetilde{\mathcal{A}} = \emptyset$
- Given a program P over $\mathcal{A}$, consider the program

$$\begin{aligned}\widetilde{P} \quad = \quad & \{r \in P \mid \mathit{head}(r) \neq {\sim}a\} \\ & \cup \{\leftarrow \mathit{body}(r) \cup \{{\sim}\widetilde{a}\} \mid r \in P \text{ and } \mathit{head}(r) = {\sim}a\} \\ & \cup \{\widetilde{a} \leftarrow {\sim}a \mid r \in P \text{ and } \mathit{head}(r) = {\sim}a\}\end{aligned}$$

- A set X of atoms is a stable model of a program P (with default negation in rule heads) over $\mathcal{A}$,
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Outline

Disjunctive logic programs

- A disjunctive rule, r, is of the form

$$a_1 \,;\ldots\,; a_m \leftarrow a_{m+1},\ldots,a_n,\sim a_{n+1},\ldots,\sim a_o$$

  where $0 \leq m \leq n \leq o$ and each a_i is an atom for $0 \leq i \leq o$
- A disjunctive logic program is a finite set of disjunctive rules
- Notation

$$\begin{aligned}
head(r) &= \{a_1,\ldots,a_m\}\\
body(r) &= \{a_{m+1},\ldots,a_n,\sim a_{n+1},\ldots,\sim a_o\}\\
body(r)^+ &= \{a_{m+1},\ldots,a_n\}\\
body(r)^- &= \{a_{n+1},\ldots,a_o\}\\
atom(P) &= \bigcup_{r\in P}\left(head(r)\cup body(r)^+\cup body(r)^-\right)\\
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Stable models

- Positive programs
  - A set X of atoms is closed under a positive program P iff for any $r \in P$, $head(r) \cap X \neq \emptyset$ whenever $body(r)^+ \subseteq X$
    - X corresponds to a model of P (seen as a formula)
  - The set of all $\subseteq$-minimal sets of atoms being closed under a positive program P is denoted by $\min_{\subseteq}(P)$
    - $\min_{\subseteq}(P)$ corresponds to the $\subseteq$-minimal models of P (ditto)
- Disjunctive programs
  - The reduct, P^X, of a disjunctive program P relative to a set X of atoms is defined by

$$P^X = \{head(r) \leftarrow body(r)^+ \mid r \in P \text{ and } body(r)^- \cap X = \emptyset\}$$

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A "positive" example

$$P = \left\{ \begin{array}{lcl} a & \leftarrow & \\ b\,;c & \leftarrow & a \end{array} \right\}$$

- The sets $\{a,b\}$, $\{a,c\}$, and $\{a,b,c\}$ are closed under P
- We have $\min_{\subseteq}(P) = \{\{a,b\},\{a,c\}\}$

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Graph coloring (reloaded)

```
node(1..6).

edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
edge(4,1;2).    edge(5,3;4;6).  edge(6,2;3;5).


color(X,r) | color(X,b) | color(X,g) :- node(X).


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More Examples

- $P_1 = \{a\,;b\,;c \leftarrow\}$
  - stable models $\{a\}$, $\{b\}$, and $\{c\}$
- $P_2 = \{a\,;b\,;c \leftarrow \;,\; \leftarrow a\}$
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- $P_3 = \{a\,;b\,;c \leftarrow\ ,\ \leftarrow a\ ,\ b \leftarrow c\ ,\ c \leftarrow b\}$
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Some properties

- A disjunctive logic program may have zero, one, or multiple stable models
- If X is a stable model of a disjunctive logic program P, then X is a model of P (seen as a formula)
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$$P = \left\{ \begin{array}{lcl} a(1,2) & \leftarrow & \\ b(X)\,;c(Y) & \leftarrow & a(X,Y), \sim c(Y) \end{array} \right\}$$

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Default negation in rule heads

- Consider disjunctive rules of the form

$$a_1 \,;\ldots\,; a_m \,; {\sim}a_{m+1} \,;\ldots\,; {\sim}a_n \leftarrow a_{n+1},\ldots,a_o,{\sim}a_{o+1},\ldots,{\sim}a_p$$

where $0 \leq m \leq n \leq o \leq p$ and each a_i is an atom for $0 \leq i \leq p$

- Given a program P over $\mathcal{A}$, consider the program

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Outline

Propositional theories

- Formulas are formed from
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  - $\bot$

  using
  - conjunction ($\wedge$)
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  - implication ($\rightarrow$)
- Notation

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Reduct

- The satisfaction relation $X \models \phi$ between a set X of atoms and a (set of) formula(s) ϕ is defined as in propositional logic
- The reduct, ϕ^X, of a formula ϕ relative to a set X of atoms is defined recursively as follows:

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- A set X of atoms is a stable model of a propositional theory Φ, if $X \in \min_{\subseteq}(\Phi^X)$
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Logic programs as propositional theories

- The normal logic program $P = \{p \leftarrow \sim q, \ q \leftarrow \sim p\}$ corresponds to $\tau[P] = \{\sim q \rightarrow p, \ \sim p \rightarrow q\}$
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Grounding: Overview

Grounding by example

$$
\begin{array}{l}
d(a)\\
d(c)\\
d(d)\\
p(a,b)\\
p(b,c)\\
p(c,d)\\
p(X,Z) \leftarrow p(X,Y), p(Y,Z)\\
q(a)\\
q(b)\\
q(X) \leftarrow \sim r(X), d(X)\\
r(X) \leftarrow \sim q(X), d(X)\\
s(X) \leftarrow \sim r(X), p(X,Y), q(Y)
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Grounding by example

Safe ?

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$d(c)$
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Grounding by example

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Match

- A **substitution** is a mapping from variables to terms
- Given sets B and D of atoms, a substitution θ is a match of B in D, if $B\theta \subseteq D$
- Given a set B of atoms and a set D of ground atoms, define

$$\Theta(B, D) = \{ \theta \mid \theta \text{ is a } \subseteq\text{-minimal match of } B \text{ in } D \}$$

- Example $\{X \mapsto 1\}$ and $\{X \mapsto 2\}$ are $\subseteq$-minimal matches of $\{p(X)\}$ in $\{p(1), p(2), p(3)\}$, while match $\{X \mapsto 1, Y \mapsto 2\}$ is not

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- Example $\{X \mapsto 1\}$ and $\{X \mapsto 2\}$ are $\subseteq$-minimal matches of $\{p(X)\}$ in $\{p(1), p(2), p(3)\}$, while match $\{X \mapsto 1, Y \mapsto 2\}$ is not

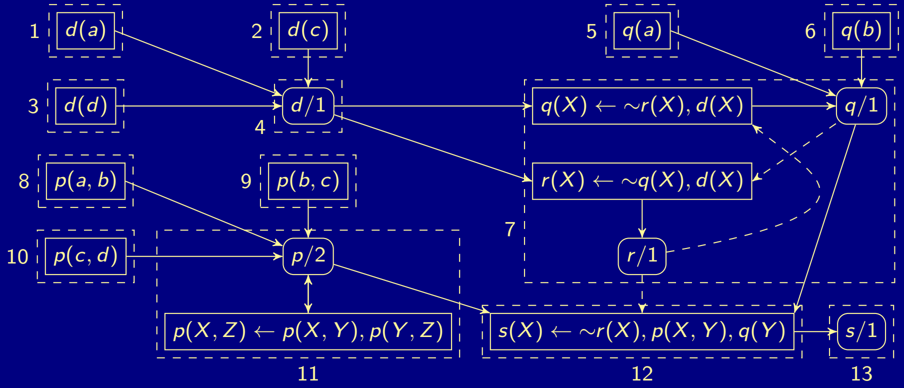
Naive instantiation

Algorithm 1: NaiveInstantiation

Input : A safe (first-order) logic program P
Output : A ground logic program P'

$D := \emptyset$
$P' := \emptyset$
repeat
    $D' := D$
    foreach $r \in P$ **do**
        $B := body(r)^+$
        foreach $\theta \in \Theta(B, D)$ **do**
            $D := D \cup \{head(r)\theta\}$
            $P' := P' \cup \{r\theta\}$
until $D = D'$

Predicate-rule dependency graph



Instantiation

SCC	$\Theta(B,D)$	D	P'
1	$\{\emptyset\}$	$d(a)$	$d(a) \leftarrow$
2	$\{\emptyset\}$	$d(c)$	$d(c) \leftarrow$
3	$\{\emptyset\}$	$d(d)$	$d(d) \leftarrow$
5	$\{\emptyset\}$	$q(a)$	$q(a) \leftarrow$
6	$\{\emptyset\}$	$q(b)$	$q(b) \leftarrow$
7	$\{\{X \mapsto a\},$		$q(a) \leftarrow \sim r(a), d(a)$
	$\{X \mapsto c\},$	$q(c)$	$q(c) \leftarrow \sim r(c), d(c)$
	$\{X \mapsto d\},$	$q(d)$	$q(d) \leftarrow \sim r(d), d(d)$
	$\{X \mapsto a\},$		$r(a) \leftarrow \sim q(a), d(a)$
	$\{X \mapsto c\},$	$r(c)$	$r(c) \leftarrow \sim q(c), d(c)$
	$\{X \mapsto d\}\}$	$r(d)$	$r(d) \leftarrow \sim q(d), d(d)$

Instantiation

SCC	$\Theta(B, D)$	D	P'
8	$\{\emptyset\}$	$p(a,b)$	$p(a,b) \leftarrow$
9	$\{\emptyset\}$	$p(b,c)$	$p(b,c) \leftarrow$
10	$\{\emptyset\}$	$p(c,d)$	$p(c,d) \leftarrow$
11	$\{\{X \mapsto a, Y \mapsto b, Z \mapsto c\},$	$p(a,c)$	$p(a,c) \leftarrow p(a,b), p(b,c)$
	$\{X \mapsto b, Y \mapsto c, Z \mapsto d\}\}$	$p(b,d)$	$p(b,d) \leftarrow p(b,c), p(c,d)$
	$\{\{X \mapsto a, Y \mapsto c, Z \mapsto d\},$	$p(a,d)$	$p(a,d) \leftarrow p(a,c), p(c,d)$
	$\{X \mapsto a, Y \mapsto b, Z \mapsto d\}\}$		$p(a,d) \leftarrow p(a,b), p(b,d)$
12	$\{\{X \mapsto a, Y \mapsto b\},$	$s(a)$	$s(a) \leftarrow {\sim}r(a), p(a,b), q(b)$
	$\{X \mapsto a, Y \mapsto c\},$		$s(a) \leftarrow {\sim}r(a), p(a,c), q(c)$
	$\{X \mapsto a, Y \mapsto d\},$		$s(a) \leftarrow {\sim}r(a), p(a,d), q(d)$
	$\{X \mapsto b, Y \mapsto c\},$	$s(b)$	$s(b) \leftarrow {\sim}r(b), p(b,c), q(c)$
	$\{X \mapsto b, Y \mapsto d\},$		$s(b) \leftarrow {\sim}r(b), p(b,d), q(d)$
	$\{X \mapsto c, Y \mapsto d\}\}$	$s(c)$	$s(c) \leftarrow {\sim}r(c), p(c,d), q(d)$

Computational Aspects: Overview

Outline

Consequence operator

- Let P be a positive program and X a set of atoms
  - The consequence operator T_P is defined as follows:

$$T_P X = \{head(r) \mid r \in P \text{ and } body(r) \subseteq X\}$$

  - Iterated applications of T_P are written as T_P^j for $j \geq 0$, where
    - $T_P^0 X = X$ and
    - $T_P^i X = T_P T_P^{i-1} X$ for $i \geq 1$
- For any positive program P, we have
  - $Cn(P) = \bigcup_{i \geq 0} T_P^i \emptyset$
  - $X \subseteq Y$ implies $T_P X \subseteq T_P Y$
  - $Cn(P)$ is the smallest fixpoint of T_P

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An example

- Consider the program

$$P = \{p \leftarrow,\ q \leftarrow,\ r \leftarrow p,\ s \leftarrow q, t,\ t \leftarrow r,\ u \leftarrow v\}$$

- We get

$$\begin{array}{lllllll}
T_P^0\emptyset & = & \emptyset & & & & \\
T_P^1\emptyset & = & \{p,q\} & = & T_P T_P^0\emptyset & = & T_P\emptyset \\
T_P^2\emptyset & = & \{p,q,r\} & = & T_P T_P^1\emptyset & = & T_P\{p,q\} \\
T_P^3\emptyset & = & \{p,q,r,t\} & = & T_P T_P^2\emptyset & = & T_P\{p,q,r\} \\
T_P^4\emptyset & = & \{p,q,r,t,s\} & = & T_P T_P^3\emptyset & = & T_P\{p,q,r,t\} \\
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\end{array}$$

- $Cn(P) = \{p,q,r,t,s\}$ is the smallest fixpoint of T_P because
  - $T_P\{p,q,r,t,s\} = \{p,q,r,t,s\}$ and
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Outline

Approximating stable models

- First Idea Approximate a stable model X by two sets of atoms L and U such that $L \subseteq X \subseteq U$
  - L and U constitute lower and upper bounds on X
  - L and $(\mathcal{A} \setminus U)$ describe a three-valued model of the program
- Observation

$$X \subseteq Y \text{ implies } P^Y \subseteq P^X \text{ implies } Cn(P^Y) \subseteq Cn(P^X)$$

- Properties Let X be a stable model of normal logic program P
  - If $L \subseteq X$, then $X \subseteq Cn(P^L)$
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Approximating stable models

- Second Idea

  repeat

  replace L **by** $L \cup Cn(P^U)$
  replace U **by** $U \cap Cn(P^L)$

  until *L and U do not change anymore*

- Observations
  - At each iteration step
    - L becomes larger (or equal)
    - U becomes smaller (or equal)
  - $L \subseteq X \subseteq U$ is invariant for every stable model X of P

  If $L \not\subseteq U$, then P has no stable model
  If $L = U$, then L is a stable model of P

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The simplistic expand algorithm

$$
\begin{array}{l}
\textbf{expand}_P(L, U) \\
\quad \textbf{repeat} \\
\quad\quad L' \leftarrow L \\
\quad\quad U' \leftarrow U \\
\quad\quad L \leftarrow L' \cup Cn(P^{U'}) \\
\quad\quad U \leftarrow U' \cap Cn(P^{L'}) \\
\quad\quad \textbf{if } L \not\subseteq U \textbf{ then return} \\
\quad \textbf{until } L = L' \text{ and } U = U'
\end{array}
$$

An example

$$P = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow a, \sim c \\ d \leftarrow b, \sim e \\ e \leftarrow \ \sim d \end{array} \right\}$$

	L'	$Cn(P^{U'})$	L	U'	$Cn(P^{L'})$	U
1	$\emptyset$	$\{a\}$	$\{a\}$	$\{a,b,c,d,e\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$
2	$\{a\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$
3	$\{a,b\}$	$\{a,b\}$	$\{a,b\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$	$\{a,b,d,e\}$

- Note We have $\{a,b\} \subseteq X$ and $(\mathcal{A} \setminus \{a,b,d,e\}) \cap X = (\{c\} \cap X) = \emptyset$ for every stable model X of P

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3	$\{a, b\}$	$\{a, b\}$	$\{a, b\}$	$\{a, b, d, e\}$	$\{a, b, d, e\}$	$\{a, b, d, e\}$

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The simplistic expand algorithm

- **expand**$_P$
  - tightens the approximation on stable models
  - is stable model preserving

Let's expand with d !

$$P = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow a, {\sim}c \\ d \leftarrow b, {\sim}e \\ e \leftarrow \ {\sim}d \end{array} \right\}$$

	L'	$Cn(P^{U'})$	L	U'	$Cn(P^{L'})$	U
1	$\{d\}$	$\{a\}$	$\{a, d\}$	$\{a, b, c, d, e\}$	$\{a, b, d\}$	$\{a, b, d\}$
2	$\{a, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$
3	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$

- Note $\{a, b, d\}$ is a stable model of P

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2	$\{a, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$
3	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$	$\{a, b, d\}$

- Note $\{a, b, d\}$ is a stable model of P

Let's expand with $\sim d$!

$$P = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow a, \sim c \\ d \leftarrow b, \sim e \\ e \leftarrow \ \sim d \end{array} \right\}$$

	L'	$Cn(P^{U'})$	L	U'	$Cn(P^{L'})$	U
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3	$\{a, b, e\}$	$\{a, b, e\}$	$\{a, b, e\}$	$\{a, b, e\}$	$\{a, b, e\}$	$\{a, b, e\}$

- Note $\{a, b, e\}$ is a stable model of P

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A simplistic solving algorithm

$solve_P(L, U)$

$(L, U) \leftarrow expand_P(L, U)$ // *propagation*

if $L \not\subseteq U$ **then failure** // *failure*

if $L = U$ **then output** L // *success*

else choose $a \in U \setminus L$ // *choice*

$solve_P(L \cup \{a\}, U)$

$solve_P(L, U \setminus \{a\})$

A simplistic solving algorithm

- Close to the approach taken by the ASP solver `smodels`, inspired by the Davis-Putman-Logemann-Loveland (DPLL) procedure
  - Backtracking search building a binary search tree
  - A node in the search tree corresponds to a three-valued interpretation
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Outline

Complexity

Let a be an atom and X be a set of atoms

- For a positive normal logic program P:
  - Deciding whether X is the stable model of P is P-complete
  - Deciding whether a is in the stable model of P is P-complete
- For a normal logic program P:
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- For a propositional theory Φ:
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Axiomatic Characterization: Overview

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Outline

Motivation

- Question Is there a propositional formula $F(P)$ such that the models of $F(P)$ correspond to the stable models of P ?

- Observation Although each atom is defined through a set of rules, each such rule provides only a sufficient condition for its head atom

- Idea The idea of program completion is to turn such implications into a definition by adding the corresponding necessary counterpart

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Program completion

Let P be a normal logic program

- The completion $CF(P)$ of P is defined as follows

$$CF(P) = \left\{ a \leftrightarrow \bigvee_{r \in P, head(r)=a} BF(body(r)) \mid a \in atom(P) \right\}$$

where

$$BF(body(r)) = \bigwedge_{a \in body(r)^+} a \wedge \bigwedge_{a \in body(r)^-} \neg a$$

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An example

$$P = \left\{ \begin{array}{l} a \leftarrow \\ b \leftarrow \sim a \\ c \leftarrow a, \sim d \\ d \leftarrow \sim c, \sim e \\ e \leftarrow b, \sim f \\ e \leftarrow e \end{array} \right\}$$

$$CF(P) = \left\{ \begin{array}{l} a \leftrightarrow \top \\ b \leftrightarrow \neg a \\ c \leftrightarrow a \land \neg d \\ d \leftrightarrow \neg c \land \neg e \\ e \leftrightarrow (b \land \neg f) \lor e \\ f \leftrightarrow \bot \end{array} \right\}$$

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A closer look

- $CF(P)$ is logically equivalent to $\overleftarrow{CF}(P) \cup \overrightarrow{CF}(P)$, where

$$\begin{aligned}
\overleftarrow{CF}(P) &= \left\{a \leftarrow \textstyle\bigvee_{B \in body_P(a)} BF(B) \mid a \in atom(P)\right\} \\
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$$body_P(a) = \{body(r) \mid r \in P \text{ and } head(r) = a\}$$

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Supported models

- Every stable model of P is a model of $CF(P)$, but not vice versa

  Models of $CF(P)$ are called the supported models of P

  In other words, every stable model of P is a supported model of P

  By definition, every supported model of P is also a model of P

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The mismatch

- Question What causes the mismatch between supported models and stable models?
- Hint Consider the unstable yet supported model $\{a, c, e\}$ of P !
- Answer Cyclic derivations are causing the mismatch between supported and stable models
  - Atoms in a stable model can be "derived" from a program in a finite number of steps
  - Atoms in a cycle (not being "supported from outside the cycle") cannot be "derived" from a program in a finite number of steps
    - But such atoms do not contradict the completion of a program and do thus not eliminate an unstable supported model

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Non-cyclic derivations

Let X be a stable model of normal logic program P

- For every atom $A \in X$, there is a finite sequence of positive rules

  $$\langle r_1, \dots, r_n \rangle$$

  such that

  1. $head(r_1) = A$
  2. $body(r_i)^+ \subseteq \{head(r_j) \mid i < j \leq n\}$ for $1 \leq i \leq n$
  3. $r_i \in P^X$ for $1 \leq i \leq n$

- That is, each atom of X has a non-cyclic derivation from P^X

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- The origin of (potential) circular derivations can be read off the positive atom dependency graph $G(P)$ of a logic program P given by

$$(atom(P), \{(a,b) \mid r \in P, a \in body(r)^+, head(r) = b\})$$

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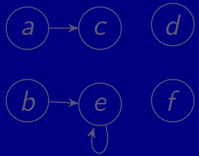

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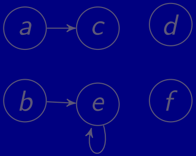

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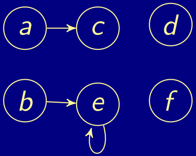

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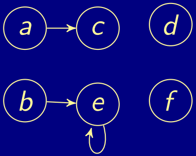

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Tight programs

- A logic program P is called tight, if $G(P)$ is acyclic
- For tight programs, stable and supported models coincide:

Fages' Theorem

Let P be a tight normal logic program and $X \subseteq atom(P)$
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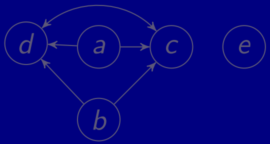

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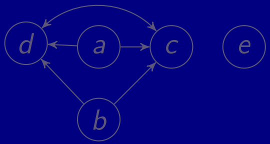

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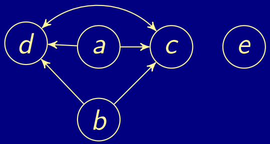

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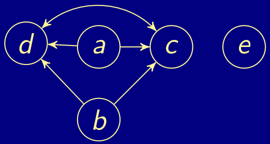

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Outline

Motivation

- Question Is there a propositional formula $F(P)$ such that the models of $F(P)$ correspond to the stable models of P ?

- Observation Starting from the completion of a program, the problem boils down to eliminating the circular support of atoms holding in the supported models of the program

- Idea Add formulas prohibiting circular support of sets of atoms

- Note Circular support between atoms a and b is possible, if a has a path to b and b has a path to a in the program's positive atom dependency graph

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Loops

Let P be a normal logic program, and
let $G(P) = (atom(P), E)$ be the positive atom dependency graph of P

- A set $\emptyset \subset L \subseteq atom(P)$ is a loop of P
  if it induces a non-trivial strongly connected subgraph of $G(P)$

  That is, each pair of atoms in L is connected by a path of non-zero length in $(L, E \cap (L \times L))$

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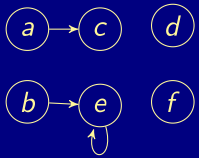

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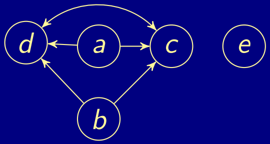

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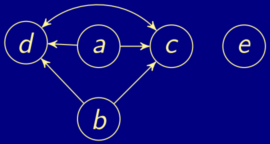

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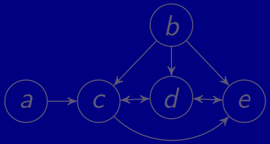

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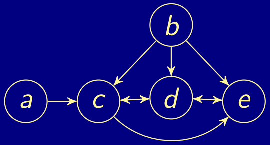

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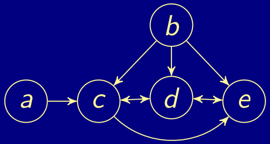

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Loop formulas

Let P be a normal logic program

- For $L \subseteq atom(P)$, define the external supports of L for P as

$$ES_P(L) = \{r \in P \mid head(r) \in L \text{ and } body(r)^+ \cap L = \emptyset\}$$

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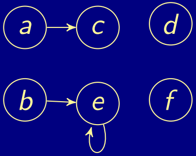

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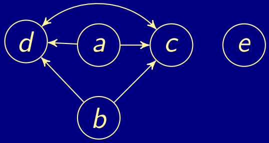

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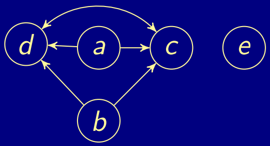

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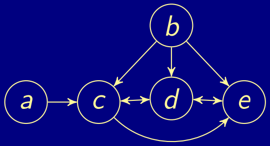

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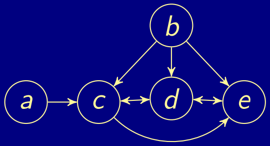

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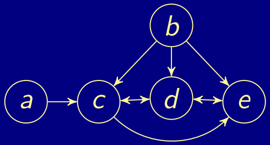

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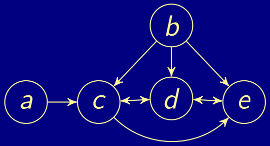

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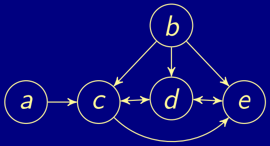

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Lin-Zhao Theorem

Theorem

Let P be a normal logic program and $X \subseteq atom(P)$
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Let X be a supported model of normal logic program P

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  - $X \models \{LF_P(U) \mid U \subseteq X\}$;
  - $X \models \{LF_P(L) \mid L \in loop(P)\}$, that is, $X \models LF(P)$;
  - $X \models \{LF_P(L) \mid L \in loop(P) \text{ and } L \subseteq X\}$
- Note If X is not a stable model of P,
  then there is a loop $L \subseteq X \setminus Cn(P^X)$ such that $X \not\models LF_P(L)$

Loops and loop formulas: Properties (ctd)

- Result If $\mathcal{P} \not\subseteq \mathcal{NC}^1/poly$,[1] then there is no translation $\mathcal{T}$ from logic programs to propositional formulas such that, for each normal logic program P, both of the following conditions hold:
  1. The propositional variables in $\mathcal{T}[P]$ are a subset of $atom(P)$
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- Note Every vocabulary-preserving translation from normal logic programs to propositional formulas must be exponential (in the worst case)

- Observations
  - Translation $CF(P) \cup LF(P)$ preserves the vocabulary of P
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Operational Characterization: Overview

Outline

Interlude: Partial interpretations

or: 3-valued interpretations

A partial interpretation maps atoms onto truth values *true*, *false*, and *unknown*

- Representation $\langle T, F\rangle$, where
  - T is the set of all *true* atoms and
  - F is the set of all *false* atoms
  - Truth of atoms in $\mathcal{A} \setminus (T \cup F)$ is *unknown*
- Properties
  - $\langle T, F\rangle$ is conflicting if $T \cap F \neq \emptyset$
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- Definition For $\langle T_1, F_1\rangle$ and $\langle T_2, F_2\rangle$, define
  - $\langle T_1, F_1\rangle \sqsubseteq \langle T_2, F_2\rangle$ iff $T_1 \subseteq T_2$ and $F_1 \subseteq F_2$
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Outline

Basic idea

- Idea Extend T_P to normal logic programs
- Logical background The idea is to turn a program's completion into an operator such that
  - the head atom of a rule must be *true*
    if the rule's body is *true*
  - an atom must be *false*
    if the body of each rule having it as head is *false*

Definition

- Let P be a normal logic program
- Define

$$\mathbf{\Phi}_P\langle T,F\rangle = \langle \mathbf{T}_P\langle T,F\rangle, \mathbf{F}_P\langle T,F\rangle\rangle$$

where

$$\mathbf{T}_P\langle T,F\rangle = \{ head(r) \mid r \in P, body(r)^+ \subseteq T, body(r)^- \subseteq F \}$$

$$\mathbf{F}_P\langle T,F\rangle = \{ a \in atom(P) \mid \\ body(r)^+ \cap F \neq \emptyset \text{ or } body(r)^- \cap T \neq \emptyset \\ \text{for each } r \in P \text{ such that } head(r) = a \}$$

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\mathbf{F}_P\langle T,F\rangle &= \{a \in atom(P) \mid \\
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\end{aligned}$$

Example

$$P = \left\{ \begin{array}{lll} a \leftarrow & c \leftarrow a, \sim d & e \leftarrow b, \sim f \\ b \leftarrow \sim a & d \leftarrow \sim c, \sim e & e \leftarrow e \end{array} \right\}$$

- Let's iterate $\mathbf{\Phi}_P$ on $\langle \{a\}, \{d\} \rangle$:

$$\begin{array}{rcl} \mathbf{\Phi}_P \langle \{a\}, \{d\} \rangle & = & \langle \{a, c\}, \{b, f\} \rangle \\ \mathbf{\Phi}_P \langle \{a, c\}, \{b, f\} \rangle & = & \langle \{a\}, \{b, d, f\} \rangle \\ \mathbf{\Phi}_P \langle \{a\}, \{b, d, f\} \rangle & = & \langle \{a, c\}, \{b, f\} \rangle \\ & \vdots & \end{array}$$

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Fitting semantics

- Define the iterative variant of $\mathbf{\Phi}_P$ analogously to T_P:

$$\mathbf{\Phi}_P^0\langle T, F\rangle = \langle T, F\rangle \qquad \mathbf{\Phi}_P^{i+1}\langle T, F\rangle = \mathbf{\Phi}_P\mathbf{\Phi}_P^i\langle T, F\rangle$$

- Define the Fitting semantics of a normal logic program P as the partial interpretation:

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Example

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Properties

Let P be a normal logic program

- $\mathbf{\Phi}_P\langle\emptyset,\emptyset\rangle$ is monotonic
  That is, $\mathbf{\Phi}^i_P\langle\emptyset,\emptyset\rangle \sqsubseteq \mathbf{\Phi}^{i+1}_P\langle\emptyset,\emptyset\rangle$
- The Fitting semantics of P is
  - not conflicting,
  - and generally not total

Fitting fixpoints

Let P be a normal logic program,
and let $\langle T, F \rangle$ be a partial interpretation

- Define $\langle T, F \rangle$ as a Fitting fixpoint of P if $\mathbf{\Phi}_P\langle T, F \rangle = \langle T, F \rangle$
  - The Fitting semantics is the $\sqsubseteq$-least Fitting fixpoint of P
  - Any other Fitting fixpoint extends the Fitting semantics
  - Total Fitting fixpoints correspond to supported models

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$$P = \left\{ \begin{array}{lll} a \leftarrow & c \leftarrow a, \sim d & e \leftarrow b, \sim f \\ b \leftarrow \sim a & d \leftarrow \sim c, \sim e & e \leftarrow e \end{array} \right\}$$

- P has three total Fitting fixpoints:
  - $\langle \{a, c\}, \{b, d, e\} \rangle$
  - $\langle \{a, d\}, \{b, c, e\} \rangle$
  - $\langle \{a, c, e\}, \{b, d\} \rangle$
- P has three supported models, two of them are stable models

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Let P be a normal logic program,
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- Let $\mathbf{\Phi}_P\langle T, F\rangle = \langle T', F'\rangle$
- If X is a stable model of P such that $T \subseteq X$ and $X \cap F = \emptyset$, then $T' \subseteq X$ and $X \cap F' = \emptyset$

  That is, $\mathbf{\Phi}_P$ is stable model preserving

  Hence, $\mathbf{\Phi}_P$ can be used for approximating stable models and so for propagation in ASP-solvers

- However, $\mathbf{\Phi}_P$ is still insufficient, because total fixpoints correspond to supported models, not necessarily stable models

  Note The problem is the same as with program completion

  The missing piece is non-circularity of derivations !

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Unfounded sets

Let P be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation

- A set $U \subseteq atom(P)$ is an unfounded set of P wrt $\langle T, F\rangle$,
if we have for each rule $r \in P$ such that $head(r) \in U$ either

   1. $body(r)^+ \cap F \neq \emptyset$ or $body(r)^- \cap T \neq \emptyset$ or
   2. $body(r)^+ \cap U \neq \emptyset$

- Intuitively, $\langle T, F\rangle$ is what we already know about P
- Rules satisfying Condition 1 are not usable for further derivations
- Condition 2 is the unfounded set condition treating cyclic derivations:
All rules still being usable to derive an atom in U require an(other) atom in U to be true

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- Intuitively, $\langle T, F\rangle$ is what we already know about P
- Rules satisfying Condition 1 are not usable for further derivations
- Condition 2 is the unfounded set condition treating cyclic derivations: All rules still being usable to derive an atom in U require an(other) atom in U to be true

Unfounded sets

Let P be a normal logic program,
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Example

$$P = \left\{ \begin{array}{lcl} a & \leftarrow & b \\ b & \leftarrow & a \end{array} \right\}$$

- $\emptyset$ is an unfounded set (by definition)
- $\{a\}$ is not an unfounded set of P wrt $\langle\emptyset,\emptyset\rangle$
- $\{a\}$ is an unfounded set of P wrt $\langle\emptyset,\{b\}\rangle$
- $\{a\}$ is not an unfounded set of P wrt $\langle\{b\},\emptyset\rangle$
- Analogously for $\{b\}$
- $\{a,b\}$ is an unfounded set of P wrt $\langle\emptyset,\emptyset\rangle$
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Greatest unfounded sets

Let P be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation

- Observation The union of two unfounded sets is an unfounded set
- The greatest unfounded set of P wrt $\langle T, F\rangle$ is the union of all unfounded sets of P wrt $\langle T, F\rangle$

  It is denoted by $\mathbf{U}_P\langle T, F\rangle$
- Alternatively, we may define

$$\mathbf{U}_P\langle T, F\rangle = atom(P) \setminus Cn(\{r \in P \mid body(r)^+ \cap F = \emptyset\}^T)$$

- Note $Cn(\{r \in P \mid body(r)^+ \cap F = \emptyset\}^T)$ contains all non-circularly derivable atoms from P wrt $\langle T, F\rangle$

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Outline

Well-founded operator

Let P be a normal logic program,
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- Observation Condition 1 (in the definition of an unfounded set) corresponds to $\mathbf{F}_P\langle T, F\rangle$ of Fitting's $\mathbf{\Phi}_P\langle T, F\rangle$
- Idea Extend (negative part of) Fitting's operator $\mathbf{\Phi}_P$
  That is,
  - keep definition of $\mathbf{T}_P\langle T, F\rangle$ from $\mathbf{\Phi}_P\langle T, F\rangle$ and
  - replace $\mathbf{F}_P\langle T, F\rangle$ from $\mathbf{\Phi}_P\langle T, F\rangle$ by $\mathbf{U}_P\langle T, F\rangle$
- In words, an atom must be *false*
  if it belongs to the greatest unfounded set
- Definition $\mathbf{\Omega}_P\langle T, F\rangle = \langle \mathbf{T}_P\langle T, F\rangle, \mathbf{U}_P\langle T, F\rangle\rangle$

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Example

$$P = \left\{ \begin{array}{lll} a \leftarrow & c \leftarrow a, \sim d & e \leftarrow b, \sim f \\ b \leftarrow \sim a & d \leftarrow \sim c, \sim e & e \leftarrow e \end{array} \right\}$$

- Let's iterate $\mathbf{\Omega}_{P_1}$ on $\langle \{c\}, \emptyset \rangle$:

$$\begin{array}{rcl} \mathbf{\Omega}_P \langle \{c\}, \emptyset \rangle & = & \langle \{a\}, \{d, f\} \rangle \\ \mathbf{\Omega}_P \langle \{a\}, \{d, f\} \rangle & = & \langle \{a, c\}, \{b, e, f\} \rangle \\ \mathbf{\Omega}_P \langle \{a, c\}, \{b, e, f\} \rangle & = & \langle \{a\}, \{b, d, e, f\} \rangle \\ \mathbf{\Omega}_P \langle \{a\}, \{b, d, e, f\} \rangle & = & \langle \{a, c\}, \{b, e, f\} \rangle \\ & \vdots & \end{array}$$

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Well-founded semantics

- Define the iterative variant of $\mathbf{\Omega}_P$ analogously to $\mathbf{\Phi}_P$:

$$\mathbf{\Omega}^0_P\langle T,F\rangle = \langle T,F\rangle \qquad \mathbf{\Omega}^{i+1}_P\langle T,F\rangle = \mathbf{\Omega}_P\mathbf{\Omega}^i_P\langle T,F\rangle$$

- Define the well-founded semantics of a normal logic program P as the partial interpretation:

$$\bigsqcup_{i\geq 0}\mathbf{\Omega}^i_P\langle\emptyset,\emptyset\rangle$$

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Properties

Let P be a normal logic program

- $\mathbf{\Omega}_P\langle\emptyset,\emptyset\rangle$ is monotonic
  That is, $\mathbf{\Omega}_P^i\langle\emptyset,\emptyset\rangle \sqsubseteq \mathbf{\Omega}_P^{i+1}\langle\emptyset,\emptyset\rangle$
- The well-founded semantics of P is
  - not conflicting,
  - and generally not total
- We have $\bigsqcup_{i\geq 0}\mathbf{\Phi}_P^i\langle\emptyset,\emptyset\rangle \sqsubseteq \bigsqcup_{i\geq 0}\mathbf{\Omega}_P^i\langle\emptyset,\emptyset\rangle$

Well-founded fixpoints

Let P be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation

- Define $\langle T, F\rangle$ as a well-founded fixpoint of P if $\mathbf{\Omega}_P\langle T, F\rangle = \langle T, F\rangle$
  - The well-founded semantics is the $\sqsubseteq$-least well-founded fixpoint of P
  - Any other well-founded fixpoint extends the well-founded semantics
  - Total well-founded fixpoints correspond to stable models

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Well-founded fixpoints: Example

$$P = \left\{ \begin{array}{lll} a \leftarrow & c \leftarrow a, \sim d & e \leftarrow b, \sim f \\ b \leftarrow \sim a & d \leftarrow \sim c, \sim e & e \leftarrow e \end{array} \right\}$$

- P has two total well-founded fixpoints:
  1. $\langle \{a, c\}, \{b, d, e\} \rangle$
  2. $\langle \{a, d\}, \{b, c, e\} \rangle$
- Both of them represent stable models

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Properties

Let P be a normal logic program,
and let $\langle T, F\rangle$ be a partial interpretation

- Let $\mathbf{\Omega}_P\langle T, F\rangle = \langle T', F'\rangle$
- If X is a stable model of P such that $T \subseteq X$ and $X \cap F = \emptyset$, then $T' \subseteq X$ and $X \cap F' = \emptyset$

  That is, $\mathbf{\Omega}_P$ is stable model preserving

  Hence, $\mathbf{\Omega}_P$ can be used for approximating stable models and so for propagation in ASP-solvers
- In contrast to $\mathbf{\Phi}_P$, operator $\mathbf{\Omega}_P$ is sufficient for propagation because total fixpoints correspond to stable models
- Note In addition to $\mathbf{\Omega}_P$, most ASP-solvers apply backward propagation, originating from program completion (although this is unnecessary from a formal point of view)

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  Hence, $\mathbf{\Omega}_P$ can be used for approximating stable models and so for propagation in ASP-solvers
- In contrast to $\mathbf{\Phi}_P$, operator $\mathbf{\Omega}_P$ is sufficient for propagation because total fixpoints correspond to stable models
- Note In addition to $\mathbf{\Omega}_P$, most ASP-solvers apply backward propagation, originating from program completion
(although this is unnecessary from a formal point of view)

Proof-theoretic Characterization: Overview

Motivation

- Goal Analyze computations in ASP solvers
- Wanted A declarative and fine-grained instrument for characterizing operations as well as strategies of ASP solvers
- Idea View stable model computations as derivations in an inference system

  Consider Tableau-based proof systems for analyzing ASP solving

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Tableau calculi

- Traditionally, tableau calculi are used for
  - automated theorem proving and
  - proof theoretical analysis

  in classical as well as non-classical logics
- General idea Given an input, prove some property by decomposition
  Decomposition is done by applying deduction rules
- For details, see *Handbook of Tableau Methods*, Kluwer, 1999

General definitions

- A tableau is a (mostly binary) tree
- A branch in a tableau is a path from the root to a leaf
- A branch containing $\gamma_1, \dots, \gamma_m$ can be extended by applying tableau rules of form

$$\frac{\gamma_1, \dots, \gamma_m}{\begin{array}{c}\alpha_1 \\ \vdots \\ \alpha_n\end{array}} \qquad\qquad \frac{\gamma_1, \dots, \gamma_m}{\beta_1 \mid \dots \mid \beta_n}$$

Rules of the former format append entries $\alpha_1, \dots, \alpha_n$ to the branch

Rules of the latter format create multiple sub-branches for $\beta_1, \dots, \beta_n$

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Example

- A simple tableau calculus for proving unsatisfiability of propositional formulas, composed from $\neg$, $\wedge$, and $\vee$, consists of rules

$$\frac{\neg\neg\alpha}{\alpha} \qquad \frac{\alpha_1 \wedge \alpha_2}{\begin{array}{c}\alpha_1\\ \alpha_2\end{array}} \qquad \frac{\beta_1 \vee \beta_2}{\beta_1 \mid \beta_2}$$

- All rules are semantically valid, when interpreting entries in a branch conjunctively and distinct (sub-)branches as connected disjunctively
- A propositional formula φ is unsatisfiable iff there is a tableau with φ as the root node such that
  1. all other entries can be produced by tableau rules and
  2. every branch contains some formulas α and $\neg\alpha$

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Example

(1)	$a \wedge ((\neg b \wedge (\neg a \vee b)) \vee \neg\neg\neg a)$	$[\varphi]$
(2)	a	[1]
(3)	$(\neg b \wedge (\neg a \vee b)) \vee \neg\neg\neg a$	[1]

(4)	$\neg b \wedge (\neg a \vee b)$	[3]	**(9)**	$\neg\neg\neg a$	[3]
(5)	$\neg b$	[4]	**(10)**	$\neg a$	[9]
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(7)	$\neg a$	[6]	**(8)**	b	[6]

All three branches of the tableau are contradictory (cf 2, 5, 7, 8, 10)

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Tableaux and ASP

- A tableau rule captures an elementary inference scheme in an ASP solver
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- An entire tableau represents a traversal of the search space

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ASP-specific definitions

- A (signed) tableau for a logic program P is a binary tree such that
  - the root node of the tree consists of the rules in P;
  - the other nodes in the tree are entries of the form $\mathbf{T}v$ or $\mathbf{F}v$, called signed literals, where v is a variable,
  - generated by extending a tableau using deduction rules (given below)
- An entry $\mathbf{T}v$ ($\mathbf{F}v$) reflects that variable v is *true* (*false*) in a corresponding variable assignment

  A set of signed literals constitutes a partial assignment
- For a normal logic program P,
  - atoms of P in $atom(P)$ and
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Tableau rules for ASP at a glance

$$(FTB)\quad \frac{p \leftarrow l_1,\dots,l_n \quad \mathbf{t}l_1,\dots,\mathbf{t}l_n}{\mathbf{T}\{l_1,\dots,l_n\}}$$

$$(BFB)\quad \frac{\mathbf{F}\{l_1,\dots,l_i,\dots,l_n\} \quad \mathbf{t}l_1,\dots,\mathbf{t}l_{i-1},\mathbf{t}l_{i+1},\dots,\mathbf{t}l_n}{\mathbf{f}l_i}$$

$$(FTA)\quad \frac{p \leftarrow l_1,\dots,l_n \quad \mathbf{T}\{l_1,\dots,l_n\}}{\mathbf{T}p}$$

$$(BFA)\quad \frac{p \leftarrow l_1,\dots,l_n \quad \mathbf{F}p}{\mathbf{F}\{l_1,\dots,l_n\}}$$

$$(FFB)\quad \frac{p \leftarrow l_1,\dots,l_i,\dots,l_n \quad \mathbf{f}l_i}{\mathbf{F}\{l_1,\dots,l_i,\dots,l_n\}}$$

$$(BTB)\quad \frac{\mathbf{T}\{l_1,\dots,l_i,\dots,l_n\}}{\mathbf{t}l_i}$$

$$(FFA)\quad \frac{\mathbf{F}B_1,\dots,\mathbf{F}B_m}{\mathbf{F}p}\quad (\S)$$

$$(BTA)\quad \frac{\mathbf{T}p \quad \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m}{\mathbf{T}B_i}\quad (\S)$$

$$(WFN)\quad \frac{\mathbf{F}B_1,\dots,\mathbf{F}B_m}{\mathbf{F}p}\quad (\dagger)$$

$$(WFJ)\quad \frac{\mathbf{T}p \quad \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m}{\mathbf{T}B_i}\quad (\dagger)$$

$$(FL)\quad \frac{\mathbf{F}B_1,\dots,\mathbf{F}B_m}{\mathbf{F}p}\quad (\ddagger)$$

$$(BL)\quad \frac{\mathbf{T}p \quad \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m}{\mathbf{T}B_i}\quad (\ddagger)$$

$$(Cut[X])\quad \frac{}{\mathbf{T}v \mid \mathbf{F}v}\quad (\sharp[X])$$

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More concepts

- A **tableau calculus** is a set of tableau rules
- A branch in a tableau is conflicting,
  if it contains both $\mathbf{T}v$ and $\mathbf{F}v$ for some variable v
- A branch in a tableau is total for a program P,
  if it contains either $\mathbf{T}v$ or $\mathbf{F}v$ for each $v \in atom(P) \cup body(P)$
- A branch in a tableau of some calculus $\mathcal{T}$ is closed,
  if no rule in $\mathcal{T}$ other than Cut can produce any new entries
- A branch in a tableau is complete,
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- A tableau is complete, if all its branches are complete
- A tableau of some calculus $\mathcal{T}$ is a refutation of $\mathcal{T}$ for a program P,
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Example

- Consider the program

$$P = \left\{ \begin{array}{l} a \leftarrow \\ c \leftarrow \sim b, \sim d \\ d \leftarrow a, \sim c \end{array} \right\}$$

having stable models $\{a, c\}$ and $\{a, d\}$

(Previewed) Example

$$a \leftarrow$$
$$c \leftarrow \sim b, \sim d$$
$$d \leftarrow a, \sim c$$

(FTB)		$\mathbf{T}\emptyset$		
(FTA)		$\mathbf{T}a$		
(FFA)		$\mathbf{F}b$		
(Cut[$atom(P)$])		$\mathbf{T}c$		$\mathbf{F}c$
	(BTA)	$\mathbf{T}\{\sim b, \sim d\}$	(BFA)	$\mathbf{F}\{\sim b, \sim d\}$
	(BTB)	$\mathbf{F}d$	(BFB)	$\mathbf{T}d$
	(FFB)	$\mathbf{F}\{a, \sim c\}$	(FTB)	$\mathbf{T}\{a, \sim c\}$

Potassco

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(FFA)		$\mathbf{F}b$		
(Cut[$atom(P)$])		$\mathbf{T}c$		$\mathbf{F}c$
	(BTA)	$\mathbf{T}\{{\sim}b, {\sim}d\}$	(BFA)	$\mathbf{F}\{{\sim}b, {\sim}d\}$
	(BTB)	$\mathbf{F}d$	(BFB)	$\mathbf{T}d$
	(FFB)	$\mathbf{F}\{a, {\sim}c\}$	(FTB)	$\mathbf{T}\{a, {\sim}c\}$

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(Previewed) Example

$$a \leftarrow$$
$$c \leftarrow \sim b, \sim d$$
$$d \leftarrow a, \sim c$$

(FTB)		$\mathbf{T}\emptyset$		
(FTA)		$\mathbf{T}a$		
(FFA)		$\mathbf{F}b$		
(Cut[$atom(P)$])		$\mathbf{T}c$		$\mathbf{F}c$
	(BTA)	$\mathbf{T}\{\sim b, \sim d\}$	(BFA)	$\mathbf{F}\{\sim b, \sim d\}$
	(BTB)	$\mathbf{F}d$	(BFB)	$\mathbf{T}d$
	(FFB)	$\mathbf{F}\{a, \sim c\}$	(FTB)	$\mathbf{T}\{a, \sim c\}$

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(Previewed) Example

$$
\begin{array}{c}
a \leftarrow \\
c \leftarrow \sim b, \sim d \\
d \leftarrow a, \sim c
\end{array}
$$

$$
\begin{array}{rcccc}
\text{(FTB)} & & & \mathbf{T}\emptyset & & \\
\text{(FTA)} & & & \mathbf{T}a & & \\
\text{(FFA)} & & & \mathbf{F}b & & \\
(\text{Cut}[atom(P)]) & & \mathbf{T}c & & & \mathbf{F}c \\
& \text{(BTA)} & \mathbf{T}\{\sim b, \sim d\} & & \text{(BFA)} & \mathbf{F}\{\sim b, \sim d\} \\
& \text{(BTB)} & \mathbf{F}d & & \text{(BFB)} & \mathbf{T}d \\
& \text{(FFB)} & \mathbf{F}\{a, \sim c\} & & \text{(FTB)} & \mathbf{T}\{a, \sim c\}
\end{array}
$$

(Previewed) Example

$$a \leftarrow$$
$$c \leftarrow \sim b, \sim d$$
$$d \leftarrow a, \sim c$$

(FTB)			$\mathbf{T}\emptyset$		
(FTA)			$\mathbf{T}a$		
(FFA)			$\mathbf{F}b$		
(Cut[$atom(P)$])		$\mathbf{T}c$			$\mathbf{F}c$
	(BTA)	$\mathbf{T}\{\sim b, \sim d\}$		(BFA)	$\mathbf{F}\{\sim b, \sim d\}$
	(BTB)	$\mathbf{F}d$		(BFB)	$\mathbf{T}d$
	(FFB)	$\mathbf{F}\{a, \sim c\}$		(FTB)	$\mathbf{T}\{a, \sim c\}$

Auxiliary definitions

- For a literal l, define conjugation functions $\mathbf{t}$ and $\mathbf{f}$ as follows

$$\mathbf{t}l = \begin{cases} \mathbf{T}l & \text{if } l \text{ is an atom} \\ \mathbf{F}a & \text{if } l = {\sim}a \text{ for an atom } a \end{cases}$$

$$\mathbf{f}l = \begin{cases} \mathbf{F}l & \text{if } l \text{ is an atom} \\ \mathbf{T}a & \text{if } l = {\sim}a \text{ for an atom } a \end{cases}$$

- Examples $\mathbf{t}a = \mathbf{T}a$, $\mathbf{f}a = \mathbf{F}a$, $\mathbf{t}{\sim}a = \mathbf{F}a$, and $\mathbf{f}{\sim}a = \mathbf{T}a$

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Auxiliary definitions

- Some tableau rules require conditions for their application
- Such conditions are specified as provisos

$$\frac{\textit{prerequisites}}{\textit{consequence}}\ (\textit{proviso}) \qquad\qquad \textit{proviso}\text{: some condition(s)}$$

- Note All tableau rules given in the sequel are stable model preserving

Forward True Body (FTB)

- Prerequisites All of a body's literals are *true*
- Consequence The body is *true*
- Tableau Rule FTB

$$\frac{\begin{array}{c} p \leftarrow l_1, \ldots, l_n \\ \mathbf{t}l_1, \ldots, \mathbf{t}l_n \end{array}}{\mathbf{T}\{l_1, \ldots, l_n\}}$$

- Example

$$\frac{\begin{array}{c} a \leftarrow b, \sim c \\ \mathbf{T}b \\ \mathbf{F}c \end{array}}{\mathbf{T}\{b, \sim c\}}$$

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- Example

$$\frac{\begin{array}{c} a \leftarrow b, \sim c \\ \mathbf{T}b \\ \mathbf{F}c \end{array}}{\mathbf{T}\{b, \sim c\}}$$

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Backward False Body (BFB)

- Prerequisites A body is *false*, and all its literals except for one are *true*
- Consequence The residual body literal is *false*
- Tableau Rule BFB

$$\frac{\begin{array}{c}\mathbf{F}\{l_1,\dots,l_i,\dots,l_n\}\\ \mathbf{t}l_1,\dots,\mathbf{t}l_{i-1},\mathbf{t}l_{i+1},\dots,\mathbf{t}l_n\end{array}}{\mathbf{f}l_i}$$

- Examples

$$\frac{\begin{array}{c}\mathbf{F}\{b,\sim c\}\\ \mathbf{T}b\end{array}}{\mathbf{T}c} \qquad \frac{\begin{array}{c}\mathbf{F}\{b,\sim c\}\\ \mathbf{F}c\end{array}}{\mathbf{F}b}$$

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- Examples

$$\frac{\begin{array}{c}\mathbf{F}\{b, \sim c\}\\ \mathbf{T}b\end{array}}{\mathbf{T}c} \qquad \frac{\begin{array}{c}\mathbf{F}\{b, \sim c\}\\ \mathbf{F}c\end{array}}{\mathbf{F}b}$$

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Forward False Body (FFB)

- Prerequisites Some literal of a body is *false*
- Consequence The body is *false*
- Tableau Rule FFB

$$\frac{p \leftarrow l_1, \dots, l_i, \dots, l_n \qquad \mathbf{f}\, l_i}{\mathbf{F}\{l_1, \dots, l_i, \dots, l_n\}}$$

- Examples

$$\frac{a \leftarrow b, \sim c \qquad \mathbf{F}b}{\mathbf{F}\{b, \sim c\}} \qquad \frac{a \leftarrow b, \sim c \qquad \mathbf{T}c}{\mathbf{F}\{b, \sim c\}}$$

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- Prerequisites Some literal of a body is *false*
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$$\frac{p \leftarrow l_1, \dots, l_i, \dots, l_n \quad \mathbf{f}\, l_i}{\mathbf{F}\{l_1, \dots, l_i, \dots, l_n\}}$$

- Examples

$$\frac{a \leftarrow b, {\sim}c \quad \mathbf{F}b}{\mathbf{F}\{b, {\sim}c\}} \qquad \frac{a \leftarrow b, {\sim}c \quad \mathbf{T}c}{\mathbf{F}\{b, {\sim}c\}}$$

Backward True Body (BTB)

- Prerequisites A body is *true*
- Consequence The body's literals are *true*
- Tableau Rule BTB

$$\frac{\mathbf{T}\{l_1,\dots,l_i,\dots,l_n\}}{\mathbf{t}l_i}$$

- Examples

$$\frac{\mathbf{T}\{b,\sim c\}}{\mathbf{T}b} \qquad \frac{\mathbf{T}\{b,\sim c\}}{\mathbf{F}c}$$

Backward True Body (BTB)

- Prerequisites A body is *true*
- Consequence The body's literals are *true*
- Tableau Rule BTB

$$\frac{\mathbf{T}\{l_1, \ldots, l_i, \ldots, l_n\}}{\mathbf{t}l_i}$$

- Examples

$$\frac{\mathbf{T}\{b, \sim c\}}{\mathbf{T}b} \qquad \frac{\mathbf{T}\{b, \sim c\}}{\mathbf{F}c}$$

Tableau rules for bodies

Consider rule body $B = \{l_1, \dots, l_n\}$

- Rules FTB and BFB amount to implication

$$l_1 \wedge \dots \wedge l_n \rightarrow B$$

- Rules FFB and BTB amount to implication

$$B \rightarrow l_1 \wedge \dots \wedge l_n$$

- Together they yield

$$B \equiv l_1 \wedge \dots \wedge l_n$$

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- Together they yield

$$B \equiv l_1 \wedge \cdots \wedge l_n$$

Forward True Atom (FTA)

- Prerequisites Some of an atom's bodies is *true*
- Consequence The atom is *true*
- Tableau Rule FTA

$$\frac{p \leftarrow l_1, \ldots, l_n \qquad \mathbf{T}\{l_1, \ldots, l_n\}}{\mathbf{T}p}$$

- Examples

$$\frac{a \leftarrow b, {\sim}c \qquad \mathbf{T}\{b, {\sim}c\}}{\mathbf{T}a} \qquad \frac{a \leftarrow d, {\sim}e \qquad \mathbf{T}\{d, {\sim}e\}}{\mathbf{T}a}$$

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- Consequence The atom is *true*
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$$\frac{\begin{array}{c} p \leftarrow l_1, \dots, l_n \\ \mathbf{T}\{l_1, \dots, l_n\} \end{array}}{\mathbf{T}p}$$

- Examples

$$\frac{\begin{array}{c} a \leftarrow b, {\sim}c \\ \mathbf{T}\{b, {\sim}c\} \end{array}}{\mathbf{T}a} \qquad \frac{\begin{array}{c} a \leftarrow d, {\sim}e \\ \mathbf{T}\{d, {\sim}e\} \end{array}}{\mathbf{T}a}$$

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Backward False Atom (BFA)

- Prerequisites An atom is *false*
- Consequence The bodies of all rules with the atom as head are *false*
- Tableau Rule BFA

$$\frac{p \leftarrow l_1, \dots, l_n \quad \mathbf{F}p}{\mathbf{F}\{l_1, \dots, l_n\}}$$

- Examples

$$\frac{a \leftarrow b, \sim c \quad \mathbf{F}a}{\mathbf{F}\{b, \sim c\}} \qquad \frac{a \leftarrow d, \sim e \quad \mathbf{F}a}{\mathbf{F}\{d, \sim e\}}$$

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Forward False Atom (FFA)

- Prerequisites For some atom, the bodies of all rules with the atom as head are *false*
- Consequence The atom is *false*
- Tableau Rule FFA

$$\frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p} \ (body_P(p) = \{B_1, \dots, B_m\})$$

- Example

$$\frac{\begin{array}{c}\mathbf{F}\{b, \sim c\} \\ \mathbf{F}\{d, \sim e\}\end{array}}{\mathbf{F}a} \ (body_P(a) = \{\{b, \sim c\}, \{d, \sim e\}\})$$

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- Example

$$\frac{\begin{array}{c}\mathbf{F}\{b,\sim c\}\\ \mathbf{F}\{d,\sim e\}\end{array}}{\mathbf{F}a}\;(\mathit{body}_P(a)=\{\{b,\sim c\},\{d,\sim e\}\})$$

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Backward True Atom (BTA)

- Prerequisites An atom is *true*, and the bodies of all rules with the atom as head except for one are *false*
- Consequence The residual body is *true*
- Tableau Rule BTA

$$\frac{\begin{array}{c}\mathbf{T}p\\ \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m\end{array}}{\mathbf{T}B_i}\ (body_P(p)=\{B_1,\dots,B_m\})$$

- Examples

$$\frac{\begin{array}{c}\mathbf{T}a\\ \mathbf{F}\{b,\sim c\}\end{array}}{\mathbf{T}\{d,\sim e\}}\ (*) \qquad \frac{\begin{array}{c}\mathbf{T}a\\ \mathbf{F}\{d,\sim e\}\end{array}}{\mathbf{T}\{b,\sim c\}}\ (*)$$

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- Examples

$$\frac{\begin{array}{c}\mathbf{T}a\\ \mathbf{F}\{b,\sim c\}\end{array}}{\mathbf{T}\{d,\sim e\}}\ (*)\qquad\qquad \frac{\begin{array}{c}\mathbf{T}a\\ \mathbf{F}\{d,\sim e\}\end{array}}{\mathbf{T}\{b,\sim c\}}\ (*)$$

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Tableau rules for atoms

Consider an atom p such that $body_P(p) = \{B_1, \ldots, B_m\}$

- Rules FTA and BFA amount to implication

  $$B_1 \vee \cdots \vee B_m \rightarrow p$$

- Rules FFA and BTA amount to implication

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- Together they yield

  $$p \equiv B_1 \vee \cdots \vee B_m$$

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Relationship with program completion

Let P be a normal logic program

- The eight tableau rules introduced so far essentially provide (straightforward) inferences from $CF(P)$

Preliminaries for unfounded sets

Let P be a normal logic program

- For $P' \subseteq P$, define the greatest unfounded set of P wrt P' as

$$\mathbf{U}_P(P') = atom(P) \setminus Cn((P')^{\emptyset})$$

- For a loop $L \in loop(P)$, define the external bodies of L as

$$EB_P(L) = \{body(r) \mid r \in P, head(r) \in L, body(r)^+ \cap L = \emptyset\}$$

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Well-Founded Negation (WFN)

- Prerequisites An atom is in the greatest unfounded set wrt rules whose bodies are *false*
- Consequence The atom is *false*
- Tableau Rule WFN

$$\frac{\mathbf{F}B_1,\dots,\mathbf{F}B_m}{\mathbf{F}p}\ (p \in \mathbf{U}_P(\{r \in P \mid body(r) \not\in \{B_1,\dots,B_m\}\}))$$

- Examples

$$\frac{\begin{array}{c} a \leftarrow \sim b \\ \mathbf{F}\{\sim b\} \end{array}}{\mathbf{F}a}\ (*) \qquad \frac{\begin{array}{c} a \leftarrow a \\ a \leftarrow \sim b \\ \mathbf{F}\{\sim b\} \end{array}}{\mathbf{F}a}\ (*)$$

$$(*) \quad a \in \mathbf{U}_P(P \setminus \{a \leftarrow \sim b\})$$

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- Examples

$$\frac{\begin{array}{c} a \leftarrow {\sim}b \\ \mathbf{F}\{{\sim}b\} \end{array}}{\mathbf{F}a} \; (*) \qquad \frac{\begin{array}{c} a \leftarrow a \\ a \leftarrow {\sim}b \\ \mathbf{F}\{{\sim}b\} \end{array}}{\mathbf{F}a} \; (*)$$

$$(*) \quad a \in \mathbf{U}_P(P \setminus \{a \leftarrow {\sim}b\})$$

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Well-Founded Justification (WFJ)

- Prerequisites A *true* atom is in the greatest unfounded set wrt rules whose bodies are *false*, if a particular body is made *false*
- Consequence The respective body is *true*
- Tableau Rule WFJ

$$\frac{\begin{array}{c}\mathbf{T}p\\ \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m\end{array}}{\mathbf{T}B_i}\ (p \in \mathbf{U}_P(\{r \in P \mid body(r) \notin \{B_1,\dots,B_m\}\}))$$

- Examples

$$\begin{array}{c} a \leftarrow {\sim}b \\ \hline \end{array}\quad \frac{\mathbf{T}a}{\mathbf{T}\{{\sim}b\}}\ (*) \qquad\qquad \begin{array}{c} a \leftarrow a \\ a \leftarrow {\sim}b \end{array}\quad \frac{\mathbf{T}a}{\mathbf{T}\{{\sim}b\}}\ (*)$$

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Well-founded tableau rules

- Tableau rules WFN and WFJ ensure non-circular support for *true* atoms
- Note
  1. WFN subsumes falsifying atoms via FFA,
  2. WFJ can be viewed as “backward propagation” for unfounded sets,
  3. WFJ subsumes backward propagation of *true* atoms via BTA

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  2. WFJ can be viewed as "backward propagation" for unfounded sets,
  3. WFJ subsumes backward propagation of *true* atoms via BTA

Relationship with well-founded operator

Let P be a normal logic program, $\langle T, F\rangle$ a partial interpretation, and $P' = \{r \in P \mid body(r)^+ \cap F = \emptyset \text{ and } body(r)^- \cap T = \emptyset\}$.

- The following conditions are equivalent
  1. $p \in \mathbf{U}_P\langle T, F\rangle$
  2. $p \in \mathbf{U}_P(P')$
- Hence, the well-founded operator $\mathbf{\Omega}$ and WFN coincide
- Note In contrast to $\mathbf{\Omega}$, WFN does not necessarily require a rule body to contain a *false* literal for the rule being inapplicable

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Forward Loop (FL)

- Prerequisites The external bodies of a loop are *false*
- Consequence The atoms in the loop are *false*
- Tableau Rule FL

$$\frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p} \ (p \in L, L \in loop(P), EB_P(L) = \{B_1, \dots, B_m\})$$

- Example

$$a \leftarrow a$$
$$a \leftarrow {\sim}b$$
$$\frac{\mathbf{F}\{{\sim}b\}}{\mathbf{F}a} \ (EB_P(\{a\}) = \{\{{\sim}b\}\})$$

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$$\frac{\mathbf{F}B_1, \dots, \mathbf{F}B_m}{\mathbf{F}p} \; (p \in L, L \in loop(P), EB_P(L) = \{B_1, \dots, B_m\})$$

- Example

$$\begin{array}{c} a \leftarrow a \\ a \leftarrow \sim b \\ \hline \end{array}$$

$$\frac{\mathbf{F}\{\sim b\}}{\mathbf{F}a} \; (EB_P(\{a\}) = \{\{\sim b\}\})$$

Backward Loop (BL)

- Prerequisites An atom of a loop is *true*, and all external bodies except for one are *false*
- Consequence The residual external body is *true*
- Tableau Rule BL

$$\frac{\begin{array}{c}\mathbf{T}p\\ \mathbf{F}B_1,\dots,\mathbf{F}B_{i-1},\mathbf{F}B_{i+1},\dots,\mathbf{F}B_m\end{array}}{\mathbf{T}B_i}\ (p \in L, L \in loop(P), EB_P(L) = \{B_1,\dots,B_m\})$$

- Example

$$\begin{array}{c}a \leftarrow a\\ a \leftarrow \sim b\end{array}$$

$$\frac{\mathbf{T}a}{\mathbf{T}\{\sim b\}}\ (EB_P(\{a\}) = \{\{\sim b\}\})$$

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- Example

$$\begin{array}{c}a\leftarrow a\\ a\leftarrow\sim b\end{array}$$

$$\frac{\mathbf{T}a}{\mathbf{T}\{\sim b\}}\;(EB_P(\{a\})=\{\{\sim b\}\})$$

Tableau rules for loops

- Tableau rules FL and BL ensure non-circular support for *true* atoms
- For a loop L such that $EB_P(L) = \{B_1, \dots, B_m\}$,
  they amount to implications of form

  $$\bigvee_{p \in L} p \rightarrow B_1 \vee \dots \vee B_m$$

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  - Impractical to precompute exponentially many loop formulas
- In practice, ASP solvers such as *smodels* and *clasp*
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    - can be viewed as an interpolation of FL
  - do not directly implement BL (and neither WFJ)
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Well-known tableau calculi

- Fitting's operator $\mathbf{\Phi}$ applies forward propagation without sophisticated unfounded set checks

$$\mathcal{T}_{\mathbf{\Phi}} = \{FTB, FTA, FFB, FFA\}$$

- Well-founded operator $\mathbf{\Omega}$ replaces negation of single atoms with negation of unfounded sets

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Tableau calculi characterizing ASP solvers

- ASP solvers combine propagation with case analysis
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\mathcal{T}_{\mathit{cmodels\text{-}1}} & = & \mathcal{T}_{\mathit{completion}} \cup \{\mathit{Cut}[\mathit{atom}(P) \cup \mathit{body}(P)]\} \\
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- SAT-based ASP solvers, *assat* and *cmodels*, incrementally add loop formulas to a program's completion
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Proof complexity

- Proof complexity is used for describing the relative efficiency of different proof systems
  - It compares proof systems based on minimal refutations
  - It is independent of heuristics
- A proof system $\mathcal{T}$ polynomially simulates a proof system $\mathcal{T}'$, if every refutation of $\mathcal{T}'$ can be polynomially mapped to a refutation of $\mathcal{T}$
  - Otherwise, $\mathcal{T}$ does not polynomially simulate $\mathcal{T}'$
- For showing that proof system $\mathcal{T}$ does not polynomially simulate $\mathcal{T}'$, we have to provide an infinite witnessing family of programs such that minimal refutations of $\mathcal{T}$ asymptotically are exponentially larger than minimal refutations of $\mathcal{T}'$
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- $\mathcal{T}_{smodels}$ restricts *Cut* to *atom*(P) and $\mathcal{T}_{noMoRe}$ to *body*(P)
  Are both approaches similar or is one of them superior to the other?
- Let $\{P_a^n\}$, $\{P_b^n\}$, and $\{P_c^n\}$ be infinite families of programs where

$$P_a^n = \left\{ \begin{array}{c} x \leftarrow {\sim}x \\ x \leftarrow a_1, b_1 \\ \vdots \\ x \leftarrow a_n, b_n \end{array} \right\} \quad P_b^n = \left\{ \begin{array}{cc} \multicolumn{2}{c}{x \leftarrow c_1, \dots, c_n, {\sim}x} \\ c_1 \leftarrow a_1 & c_1 \leftarrow b_1 \\ \vdots & \vdots \\ c_n \leftarrow a_n & c_n \leftarrow b_n \end{array} \right\} \quad P_c^n = \left\{ \begin{array}{c} a_1 \leftarrow {\sim}b_1 \\ b_1 \leftarrow {\sim}a_1 \\ \vdots \\ a_n \leftarrow {\sim}b_n \\ b_n \leftarrow {\sim}a_n \end{array} \right\}$$

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$$P_a^n = \left\{ \begin{array}{c} x \leftarrow {\sim}x \\ x \leftarrow a_1, b_1 \\ \vdots \\ x \leftarrow a_n, b_n \end{array} \right\} \quad P_b^n = \left\{ \begin{array}{cc} \multicolumn{2}{c}{x \leftarrow c_1, \dots, c_n, {\sim}x} \\ c_1 \leftarrow a_1 & c_1 \leftarrow b_1 \\ \vdots & \vdots \\ c_n \leftarrow a_n & c_n \leftarrow b_n \end{array} \right\} \quad P_c^n = \left\{ \begin{array}{c} a_1 \leftarrow {\sim}b_1 \\ b_1 \leftarrow {\sim}a_1 \\ \vdots \\ a_n \leftarrow {\sim}b_n \\ b_n \leftarrow {\sim}a_n \end{array} \right\}$$

- In minimal refutations for $P_a^n \cup P_c^n$, the number of applications of $Cut[body(P_a^n \cup P_c^n)]$ with $\mathcal{T}_{noMoRe}$ is linear in n, whereas $\mathcal{T}_{smodels}$ requires exponentially many applications of $Cut[atom(P_a^n \cup P_c^n)]$
- Vice versa, minimal refutations for $P_b^n \cup P_c^n$ require linearly many applications of $Cut[atom(P_b^n \cup P_c^n)]$ with $\mathcal{T}_{smodels}$ and exponentially many applications of $Cut[body(P_b^n \cup P_c^n)]$ with $\mathcal{T}_{noMoRe}$

$\mathcal{T}_{smodels}$ versus $\mathcal{T}_{noMoRe}$

- $\mathcal{T}_{smodels}$ restricts Cut to $atom(P)$ and $\mathcal{T}_{noMoRe}$ to $body(P)$
  Are both approaches similar or is one of them superior to the other?

- Let $\{P_a^n\}$, $\{P_b^n\}$, and $\{P_c^n\}$ be infinite families of programs where

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Relative efficiency

- As witnessed by $\{P_a^n \cup P_c^n\}$ and $\{P_b^n \cup P_c^n\}$, respectively, $\mathcal{T}_{smodels}$ and $\mathcal{T}_{noMoRe}$ do not polynomially simulate one another
- Any refutation of $\mathcal{T}_{smodels}$ or $\mathcal{T}_{noMoRe}$ is a refutation of $\mathcal{T}_{nomore^{++}}$ (but not vice versa)
- Hence
  - both $\mathcal{T}_{smodels}$ and $\mathcal{T}_{noMoRe}$ are polynomially simulated by $\mathcal{T}_{nomore^{++}}$ and
  - $\mathcal{T}_{nomore^{++}}$ is polynomially simulated by neither $\mathcal{T}_{smodels}$ nor $\mathcal{T}_{noMoRe}$
- More generally, the proof system obtained with $Cut[atom(P) \cup body(P)]$ is exponentially stronger than the ones with either $Cut[atom(P)]$ or $Cut[body(P)]$
- Case analyses (at least) on atoms and bodies are mandatory in powerful ASP solvers

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Potassco

$\mathcal{T}_{smodels}$: Example tableau

$$
\begin{array}{llllll}
(r_1) & a \leftarrow \sim b & (r_2) & b \leftarrow d, \sim a & (r_3) & c \leftarrow b, d \\
(r_4) & c \leftarrow g & (r_5) & d \leftarrow c & (r_6) & d \leftarrow g \\
(r_7) & e \leftarrow f, \sim c & (r_8) & f \leftarrow \sim g & (r_9) & g \leftarrow \sim a, \sim f
\end{array}
$$

(1)	$\mathbf{T}a$	$[Cut]$
(2)	$\mathbf{T}\{\sim b\}$	$[BTA\colon r_1, 1]$
(3)	$\mathbf{F}b$	$[BTB\colon 2]$
(4)	$\mathbf{F}\{d, \sim a\}$	$[BFA\colon r_2, 3]$
(5)	$\mathbf{F}\{\sim a, \sim f\}$	$[FFB\colon r_9, 1]$
(6)	$\mathbf{F}g$	$[FFA\colon r_9, 5]$
(7)	$\mathbf{T}\{\sim g\}$	$[FTB\colon r_8, 6]$
(8)	$\mathbf{T}f$	$[FTA\colon r_8, 7]$
(9)	$\mathbf{F}\{b, d\}$	$[FFB\colon r_3, 3]$
(10)	$\mathbf{F}\{g\}$	$[FFB\colon r_4, r_6, 6]$
(11)	$\mathbf{F}c$	$[FFA\colon r_3, r_4, 9, 10]$
(12)	$\mathbf{F}\{c\}$	$[FFB\colon r_5, 11]$
(13)	$\mathbf{F}d$	$[FFA\colon r_5, r_6, 10, 12]$
(14)	$\mathbf{T}\{f, \sim c\}$	$[FTB\colon r_7, 8, 11]$
(15)	$\mathbf{T}e$	$[FTA\colon r_7, 14]$

(16)	$\mathbf{F}a$	$[Cut]$
(17)	$\mathbf{F}\{\sim b\}$	$[BFA\colon r_1, 16]$
(18)	$\mathbf{T}b$	$[BFB\colon 17]$
(19)	$\mathbf{T}\{d, \sim a\}$	$[BTA\colon r_2, 18]$
(20)	$\mathbf{T}d$	$[BTB\colon 19]$
(21)	$\mathbf{T}\{b, d\}$	$[FTB\colon r_3, 18, 20]$
(22)	$\mathbf{T}c$	$[FTA\colon r_3, 21]$
(23)	$\mathbf{F}\{f, \sim c\}$	$[FFB\colon r_7, 22]$
(24)	$\mathbf{F}e$	$[FFA\colon r_7, 23]$
(25)	$\mathbf{T}\{c\}$	$[FTB\colon r_5, 22]$

(26)	$\mathbf{T}f$	$[Cut]$
(27)	$\mathbf{F}\{\sim a, \sim f\}$	$[FFB\colon r_9, 26]$
(28)	$\mathbf{F}c$	$[WFN\colon 27]$

(29)	$\mathbf{F}f$	$[Cut]$
(30)	$\mathbf{T}\{\sim a, \sim f\}$	$[FTB\colon r_9, 16, 29]$
(31)	$\mathbf{T}g$	$[FTA\colon r_9, 30]$
(32)	$\mathbf{T}\{g\}$	$[FTB\colon r_4, r_6, 31]$
(33)	$\mathbf{F}\{\sim g\}$	$[FFB\colon r_8, 31]$

Potassco

$\mathcal{T}_{noMoRe}$: Example tableau

(r_1)	$a \leftarrow \sim b$	(r_2)	$b \leftarrow d, \sim a$	(r_3)	$c \leftarrow b, d$
(r_4)	$c \leftarrow g$	(r_5)	$d \leftarrow c$	(r_6)	$d \leftarrow g$
(r_7)	$e \leftarrow f, \sim c$	(r_8)	$f \leftarrow \sim g$	(r_9)	$g \leftarrow \sim a, \sim f$

(1)	$\mathbf{T}\{\sim b\}$	$[Cut]$
(2)	$\mathbf{T}a$	$[FTA\colon r_1, 1]$
(3)	$\mathbf{F}b$	$[BTB\colon 1]$
(4)	$\mathbf{F}\{d, \sim a\}$	$[BFA\colon r_2, 3]$
(5)	$\mathbf{F}\{\sim a, \sim f\}$	$[FFB\colon r_9, 2]$
(6)	$\mathbf{F}g$	$[FFA\colon r_9, 5]$
(7)	$\mathbf{T}\{\sim g\}$	$[FTB\colon r_8, 6]$
(8)	$\mathbf{T}f$	$[FTA\colon r_8, 7]$
(9)	$\mathbf{F}\{b, d\}$	$[FFB\colon r_3, 3]$
(10)	$\mathbf{F}\{g\}$	$[FFB\colon r_4, r_6, 6]$
(11)	$\mathbf{F}c$	$[FFA\colon r_3, r_4, 9, 10]$
(12)	$\mathbf{F}\{c\}$	$[FFB\colon r_5, 11]$
(13)	$\mathbf{F}d$	$[FFA\colon r_5, r_6, 10, 12]$
(14)	$\mathbf{T}\{f, \sim c\}$	$[FTB\colon r_7, 8, 11]$
(15)	$\mathbf{T}e$	$[FTA\colon r_7, 14]$

(16)	$\mathbf{F}\{\sim b\}$	$[Cut]$
(17)	$\mathbf{F}a$	$[FFA\colon r_1, 16]$
(18)	$\mathbf{T}b$	$[BFB\colon 16]$
(19)	$\mathbf{T}\{d, \sim a\}$	$[BTA\colon r_2, 18]$
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(22)	$\mathbf{T}c$	$[FTA\colon r_3, 21]$
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(25)	$\mathbf{T}\{c\}$	$[FTB\colon r_5, 22]$

(26)	$\mathbf{T}\{\sim g\}$	$[Cut]$
(27)	$\mathbf{F}g$	$[BTB\colon 26]$
(28)	$\mathbf{F}\{g\}$	$[FFB\colon r_4, r_6, 27]$
(29)	$\mathbf{F}c$	$[WFN\colon 28]$

(30)	$\mathbf{F}\{\sim g\}$	$[Cut]$
(31)	$\mathbf{T}g$	$[BFB\colon 30]$
(32)	$\mathbf{T}\{g\}$	$[FTB\colon r_4, r_6, 31]$
(33)	$\mathbf{F}f$	$[FFA\colon r_8, 30]$
(34)	$\mathbf{T}\{\sim a, \sim f\}$	$[FTB\colon r_9, 17, 33]$

$\mathcal{T}_{nomore^{++}}$: Example tableau

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\begin{array}{llllll}
(r_1) & a \leftarrow {\sim}b & (r_2) & b \leftarrow d, {\sim}a & (r_3) & c \leftarrow b, d \\
(r_4) & c \leftarrow g & (r_5) & d \leftarrow c & (r_6) & d \leftarrow g \\
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\end{array}
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(1)	$\mathbf{T}a$	[*Cut*]
(2)	$\mathbf{T}\{{\sim}b\}$	[*BTA*: r_1, 1]
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(13)	$\mathbf{F}d$	[*FFA*: r_5, r_6, 10, 12]
(14)	$\mathbf{T}\{f, {\sim}c\}$	[*FTB*: r_7, 8, 11]
(15)	$\mathbf{T}e$	[*FTA*: r_7, 14]

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(17)	$\mathbf{F}\{{\sim}b\}$	[*BFA*: r_1, 16]
(18)	$\mathbf{T}b$	[*BFB*: 17]
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Conflict-driven ASP Solving: Overview

Outline

Motivation

- Goal Approach to computing stable models of logic programs, based on concepts from
  - Constraint Processing (CP) and
  - Satisfiability Testing (SAT)
- Idea View inferences in ASP as unit propagation on nogoods
- Benefits
  - A uniform constraint-based framework for different kinds of inferences in ASP
  - Advanced techniques from the areas of CP and SAT
  - Highly competitive implementation

Outline

Assignments

- An assignment A over $dom(A) = atom(P) \cup body(P)$ is a sequence

$$(\sigma_1, \dots, \sigma_n)$$

  of signed literals σ_i of form $\mathbf{T}v$ or $\mathbf{F}v$ for $v \in dom(A)$ and $1 \leq i \leq n$
- $\mathbf{T}v$ expresses that v is *true* and $\mathbf{F}v$ that it is *false*
- The complement, $\overline{\sigma}$, of a literal σ is defined as $\overline{\mathbf{T}v} = \mathbf{F}v$ and $\overline{\mathbf{F}v} = \mathbf{T}v$
- $A \circ \sigma$ stands for the result of appending σ to A
- Given $A = (\sigma_1, \dots, \sigma_{k-1}, \sigma_k, \dots, \sigma_n)$, we let $A[\sigma_k] = (\sigma_1, \dots, \sigma_{k-1})$
- We sometimes identify an assignment with the set of its literals
- Given this, we access *true* and *false* propositions in A via

$$A^{\mathbf{T}} = \{v \in dom(A) \mid \mathbf{T}v \in A\} \text{ and } A^{\mathbf{F}} = \{v \in dom(A) \mid \mathbf{F}v \in A\}$$

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Nogoods, solutions, and unit propagation

- A nogood is a set $\{\sigma_1, \dots, \sigma_n\}$ of signed literals, expressing a constraint violated by any assignment containing $\sigma_1, \dots, \sigma_n$
- An assignment A such that $A^{\mathbf{T}} \cup A^{\mathbf{F}} = dom(A)$ and $A^{\mathbf{T}} \cap A^{\mathbf{F}} = \emptyset$ is a solution for a set Δ of nogoods, if $\delta \not\subseteq A$ for all $\delta \in \Delta$
- For a nogood δ, a literal $\sigma \in \delta$, and an assignment A, we say that $\overline{\sigma}$ is unit-resulting for δ wrt A, if
  1. $\delta \setminus A = \{\sigma\}$ and
  2. $\overline{\sigma} \notin A$
- For a set Δ of nogoods and an assignment A, unit propagation is the iterated process of extending A with unit-resulting literals until no further literal is unit-resulting for any nogood in Δ

Nogoods, solutions, and unit propagation

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Nogoods, solutions, and unit propagation

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Nogoods, solutions, and unit propagation

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Outline

Outline

Nogoods from logic programs
via program completion

The completion of a logic program P can be defined as follows:

$$\begin{array}{ll} & \{v_B \leftrightarrow a_1 \wedge \dots \wedge a_m \wedge \neg a_{m+1} \wedge \dots \wedge \neg a_n \mid \\ & \qquad B \in \mathit{body}(P) \text{ and } B = \{a_1, \dots, a_m, {\sim}a_{m+1}, \dots, {\sim}a_n\}\} \\ \cup & \{a \leftrightarrow v_{B_1} \vee \dots \vee v_{B_k} \mid \\ & \qquad a \in \mathit{atom}(P) \text{ and } \mathit{body}_P(a) = \{B_1, \dots, B_k\}\} \ , \end{array}$$

where $\mathit{body}_P(a) = \{\mathit{body}(r) \mid r \in P \text{ and } \mathit{head}(r) = a\}$

Nogoods from logic programs
via program completion

- The (body-oriented) equivalence

$$v_B \leftrightarrow a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n$$

can be decomposed into two implications:

Nogoods from logic programs
via program completion

- The (body-oriented) equivalence

$$v_B \leftrightarrow a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n$$

can be decomposed into two implications:

1. $v_B \rightarrow a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n$

   is equivalent to the conjunction of

$$\neg v_B \vee a_1, \ \ldots, \ \neg v_B \vee a_m, \ \neg v_B \vee \neg a_{m+1}, \ \ldots, \ \neg v_B \vee \neg a_n$$

   and induces the set of nogoods

$$\Delta(B) = \{ \{\mathbf{T}B, \mathbf{F}a_1\}, \ldots, \{\mathbf{T}B, \mathbf{F}a_m\}, \{\mathbf{T}B, \mathbf{T}a_{m+1}\}, \ldots, \{\mathbf{T}B, \mathbf{T}a_n\} \}$$

Nogoods from logic programs
via program completion

- The (body-oriented) equivalence

$$v_B \leftrightarrow a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n$$

can be decomposed into two implications:

2. $a_1 \wedge \cdots \wedge a_m \wedge \neg a_{m+1} \wedge \cdots \wedge \neg a_n \rightarrow v_B$

   gives rise to the nogood

$$\delta(B) = \{\mathbf{F}B, \mathbf{T}a_1, \ldots, \mathbf{T}a_m, \mathbf{F}a_{m+1}, \ldots, \mathbf{F}a_n\}$$

Nogoods from logic programs
via program completion

- Analogously, the (atom-oriented) equivalence

$$a \leftrightarrow v_{B_1} \vee \cdots \vee v_{B_k}$$

yields the nogoods

1. $\Delta(a) = \{\, \{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$ and
2. $\delta(a) = \{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\}$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

$\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
$\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

$$\boxed{\begin{array}{lcl} x & \leftarrow & y \\ x & \leftarrow & \sim z \end{array}} \qquad \begin{array}{l} \{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\} \\ \{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\} \end{array}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal
$\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
$\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

$$\boxed{\begin{array}{lll} x & \leftarrow & y \\ x & \leftarrow & \sim z \end{array}}$$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

$$\boxed{\begin{array}{lcl} x & \leftarrow & y \\ x & \leftarrow & \sim z \end{array}}$$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

$$\boxed{\begin{array}{lcl} x & \leftarrow & y \\ x & \leftarrow & \sim z \end{array}}$$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\} \quad \text{and} \quad \{\{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\}\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

$$\boxed{\begin{array}{lll} x & \leftarrow & y \\ x & \leftarrow & \sim z \end{array}} \qquad \begin{array}{l} \{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\} \\ \{\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\} \end{array}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal
- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \ldots, \mathbf{F}B_k\} \quad \text{and} \quad \{\{\mathbf{F}a, \mathbf{T}B_1\}, \ldots, \{\mathbf{F}a, \mathbf{T}B_k\}\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

$$\boxed{\begin{array}{lll} x & \leftarrow & y \\ x & \leftarrow & \sim z \end{array}}$$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

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For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

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- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

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- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

$$\boxed{\begin{array}{lll} x & \leftarrow & y \\ x & \leftarrow & \sim z \end{array}}$$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

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- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

$$\boxed{\begin{array}{lll} x & \leftarrow & y \\ x & \leftarrow & \sim z \end{array}}$$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \ldots, B_k\}$, we get

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$$\boxed{\begin{array}{lll} x & \leftarrow & y \\ x & \leftarrow & \sim z \end{array}}$$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

x	$\leftarrow$	y
x	$\leftarrow$	$\sim z$

$$\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$$
$$\{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

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Nogoods from logic programs
atom-oriented nogoods

- For an atom a where $body_P(a) = \{B_1, \dots, B_k\}$, we get

$$\{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\} \quad \text{and} \quad \{\,\{\mathbf{F}a, \mathbf{T}B_1\}, \dots, \{\mathbf{F}a, \mathbf{T}B_k\}\,\}$$

- Example Given Atom x with $body(x) = \{\{y\}, \{\sim z\}\}$, we obtain

$$\boxed{\begin{array}{lll} x & \leftarrow & y \\ x & \leftarrow & \sim z \end{array}} \qquad \begin{array}{l} \{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\} \\ \{\,\{\mathbf{F}x, \mathbf{T}\{y\}\}, \{\mathbf{F}x, \mathbf{T}\{\sim z\}\}\,\} \end{array}$$

For nogood $\{\mathbf{T}x, \mathbf{F}\{y\}, \mathbf{F}\{\sim z\}\}$, the signed literal

- $\mathbf{F}x$ is unit-resulting wrt assignment $(\mathbf{F}\{y\}, \mathbf{F}\{\sim z\})$ and
- $\mathbf{T}\{\sim z\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}\{y\})$

Nogoods from logic programs
body-oriented nogoods

- For a body $B = \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\}$, we get

$$\{\mathbf{F}B, \mathbf{T}a_1, \dots, \mathbf{T}a_m, \mathbf{F}a_{m+1}, \dots, \mathbf{F}a_n\}$$
$$\{\,\{\mathbf{T}B, \mathbf{F}a_1\}, \dots, \{\mathbf{T}B, \mathbf{F}a_m\}, \{\mathbf{T}B, \mathbf{T}a_{m+1}\}, \dots, \{\mathbf{T}B, \mathbf{T}a_n\}\,\}$$

- Example Given Body $\{x, \sim y\}$, we obtain

$$\boxed{\begin{array}{c} \dots \leftarrow x, \sim y \\ \vdots \\ \dots \leftarrow x, \sim y \end{array}}$$

$$\{\mathbf{F}\{x, \sim y\}, \mathbf{T}x, \mathbf{F}y\}$$
$$\{\,\{\mathbf{T}\{x, \sim y\}, \mathbf{F}x\}, \{\mathbf{T}\{x, \sim y\}, \mathbf{T}y\}\,\}$$

For nogood $\delta(\{x, \sim y\}) = \{\mathbf{F}\{x, \sim y\}, \mathbf{T}x, \mathbf{F}y\}$, the signed literal

- $\mathbf{T}\{x, \sim y\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}y)$ and
- $\mathbf{T}y$ is unit-resulting wrt assignment $(\mathbf{F}\{x, \sim y\}, \mathbf{T}x)$

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Nogoods from logic programs
body-oriented nogoods

- For a body $B = \{a_1, \dots, a_m, {\sim}a_{m+1}, \dots, {\sim}a_n\}$, we get

$$\{\mathbf{F}B, \mathbf{T}a_1, \dots, \mathbf{T}a_m, \mathbf{F}a_{m+1}, \dots, \mathbf{F}a_n\}$$
$$\{\,\{\mathbf{T}B, \mathbf{F}a_1\}, \dots, \{\mathbf{T}B, \mathbf{F}a_m\}, \{\mathbf{T}B, \mathbf{T}a_{m+1}\}, \dots, \{\mathbf{T}B, \mathbf{T}a_n\}\,\}$$

- Example Given Body $\{x, {\sim}y\}$, we obtain

$$\boxed{\begin{array}{l} \dots \leftarrow x, {\sim}y \\ \qquad\vdots \\ \dots \leftarrow x, {\sim}y \end{array}}$$

$$\{\mathbf{F}\{x, {\sim}y\}, \mathbf{T}x, \mathbf{F}y\}$$
$$\{\,\{\mathbf{T}\{x, {\sim}y\}, \mathbf{F}x\}, \{\mathbf{T}\{x, {\sim}y\}, \mathbf{T}y\}\,\}$$

For nogood $\delta(\{x, {\sim}y\}) = \{\mathbf{F}\{x, {\sim}y\}, \mathbf{T}x, \mathbf{F}y\}$, the signed literal

- $\mathbf{T}\{x, {\sim}y\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}y)$ and
- $\mathbf{T}y$ is unit-resulting wrt assignment $(\mathbf{F}\{x, {\sim}y\}, \mathbf{T}x)$

Nogoods from logic programs
body-oriented nogoods

- For a body $B = \{a_1, \dots, a_m, \sim a_{m+1}, \dots, \sim a_n\}$, we get

$$\{\mathbf{F}B, \mathbf{T}a_1, \dots, \mathbf{T}a_m, \mathbf{F}a_{m+1}, \dots, \mathbf{F}a_n\}$$
$$\{\,\{\mathbf{T}B, \mathbf{F}a_1\}, \dots, \{\mathbf{T}B, \mathbf{F}a_m\}, \{\mathbf{T}B, \mathbf{T}a_{m+1}\}, \dots, \{\mathbf{T}B, \mathbf{T}a_n\}\,\}$$

- Example Given Body $\{x, \sim y\}$, we obtain

$$\boxed{\begin{array}{r} \dots \leftarrow x, \sim y \\ \vdots \\ \dots \leftarrow x, \sim y \end{array}}$$

$$\{\mathbf{F}\{x, \sim y\}, \mathbf{T}x, \mathbf{F}y\}$$
$$\{\,\{\mathbf{T}\{x, \sim y\}, \mathbf{F}x\}, \{\mathbf{T}\{x, \sim y\}, \mathbf{T}y\}\,\}$$

For nogood $\delta(\{x, \sim y\}) = \{\mathbf{F}\{x, \sim y\}, \mathbf{T}x, \mathbf{F}y\}$, the signed literal

- $\mathbf{T}\{x, \sim y\}$ is unit-resulting wrt assignment $(\mathbf{T}x, \mathbf{F}y)$ and
- $\mathbf{T}y$ is unit-resulting wrt assignment $(\mathbf{F}\{x, \sim y\}, \mathbf{T}x)$

Characterization of stable models

for tight logic programs

Let P be a logic program and

$$\begin{array}{rcl} \Delta_P & = & \{\delta(a) \mid a \in atom(P)\} \cup \{\delta \in \Delta(a) \mid a \in atom(P)\} \\ & \cup & \{\delta(B) \mid B \in body(P)\} \cup \{\delta \in \Delta(B) \mid B \in body(P)\} \end{array}$$

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Let P be a tight logic program. Then,
$X \subseteq atom(P)$ is a stable model of P iff
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for tight logic programs, ie. free of positive recursion

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Outline

Nogoods from logic programs
via loop formulas

Let P be a normal logic program and recall that:

- For $L \subseteq atom(P)$, the external supports of L for P are

$$ES_P(L) \quad = \quad \{r \in P \mid head(r) \in L \text{ and } body(r)^+ \cap L = \emptyset\}$$

- The (disjunctive) loop formula of L for P is

$$\begin{aligned} LF_P(L) \quad &= \quad (\bigvee_{A \in L} A) \rightarrow (\bigvee_{r \in ES_P(L)} body(r)) \\ &\equiv \quad (\bigwedge_{r \in ES_P(L)} \neg body(r)) \rightarrow (\bigwedge_{A \in L} \neg A) \end{aligned}$$

  - Note The loop formula of L enforces all atoms in L to be *false* whenever L is not externally supported

- The external bodies of L for P are

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Nogoods from logic programs
loop nogoods

- For a logic program P and some $\emptyset \subset U \subseteq atom(P)$, define the loop nogood of an atom $a \in U$ as

$$\lambda(a, U) \quad = \quad \{\mathbf{T}a, \mathbf{F}B_1, \dots, \mathbf{F}B_k\}$$

where $EB_P(U) = \{B_1, \dots, B_k\}$

- We get the following set of loop nogoods for P:

$$\Lambda_P \quad = \quad \bigcup_{\emptyset \subset U \subseteq atom(P)} \{\lambda(a, U) \mid a \in U\}$$

- The set Λ_P of loop nogoods denies cyclic support among *true* atoms

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Example

- Consider the program

$$\left\{ \begin{array}{ll} x \leftarrow \sim y & u \leftarrow x \\ y \leftarrow \sim x & u \leftarrow v \\ & v \leftarrow u, y \end{array} \right\}$$

- For u in the set $\{u, v\}$, we obtain the loop nogood:

$$\lambda(u, \{u, v\}) \quad = \quad \{\mathbf{T}u, \mathbf{F}\{x\}\}$$

Similarly for v in $\{u, v\}$, we get:

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Characterization of stable models

Theorem

Let P be a logic program. Then,

$X \subseteq atom(P)$ *is a stable model of* P *iff*

$X = A^{\mathbf{T}} \cap atom(P)$ *for a (unique) solution* A *for* $\Delta_P \cup \Lambda_P$

Some remarks

- Nogoods in Λ_P augment Δ_P with conditions checking for unfounded sets, in particular, those being loops
- While $|\Delta_P|$ is linear in the size of P, Λ_P may contain exponentially many (non-redundant) loop nogoods

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Outline

Towards conflict-driven search

Boolean constraint solving algorithms pioneered for SAT led to:

- Traditional DPLL-style approach
  (DPLL stands for 'Davis-Putnam-Logemann-Loveland')
  - (Unit) propagation
  - (Chronological) backtracking
  - in ASP, eg *smodels*
- Modern CDCL-style approach
  (CDCL stands for 'Conflict-Driven Constraint Learning')
  - (Unit) propagation
  - Conflict analysis (via resolution)
  - Learning + Backjumping + Assertion
  - in ASP, eg *clasp*

DPLL-style solving

loop
- *propagate* // deterministically assign literals
- **if** no conflict **then**
  - **if** all variables assigned **then return** solution
  - **else** *decide* // non-deterministically assign some literal
- **else**
  - **if** top-level conflict **then return** unsatisfiable
  - **else**
    - *backtrack* // unassign literals propagated after last decision
    - *flip* // assign complement of last decision literal

CDCL-style solving

loop

propagate // deterministically assign literals

if no conflict **then**

if all variables assigned **then return** solution

else *decide* // non-deterministically assign some literal

else

if top-level conflict **then return** unsatisfiable

else

analyze // analyze conflict and add conflict constraint

backjump // unassign literals until conflict constraint is unit

Outline

Outline of CDNL-ASP algorithm

- Keep track of deterministic consequences by unit propagation on:
  - Program completion [Δ_P]
  - Loop nogoods, determined and recorded on demand [Λ_P]
  - Dynamic nogoods, derived from conflicts and unfounded sets [∇]
- When a nogood in $\Delta_P \cup \nabla$ becomes violated:
  - Analyze the conflict by resolution (until reaching a Unique Implication Point, short: UIP)
  - Learn the derived conflict nogood δ
  - Backjump to the earliest (heuristic) choice such that the complement of the UIP is unit-resulting for δ
  - Assert the complement of the UIP and proceed (by unit propagation)
- Terminate when either:
  - Finding a stable model (a solution for $\Delta_P \cup \Lambda_P$)
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Algorithm 2: CDNL-ASP

Input : A normal program P
Output : A stable model of P or "no stable model"

$A := \emptyset$ // *assignment over atom(P) ∪ body(P)*
$\nabla := \emptyset$ // *set of recorded nogoods*
$dl := 0$ // *decision level*

loop
 $(A, \nabla) := \textsc{NogoodPropagation}(P, \nabla, A)$
 if $\varepsilon \subseteq A$ **for some** $\varepsilon \in \Delta_P \cup \nabla$ **then** // *conflict*
  if $\max(\{dlevel(\sigma) \mid \sigma \in \varepsilon\} \cup \{0\}) = 0$ **then return** no stable model
  $(\delta, dl) := \textsc{ConflictAnalysis}(\varepsilon, P, \nabla, A)$
  $\nabla := \nabla \cup \{\delta\}$ // *(temporarily) record conflict nogood*
  $A := A \setminus \{\sigma \in A \mid dl < dlevel(\sigma)\}$ // *backjumping*
 else if $A^{\mathbf{T}} \cup A^{\mathbf{F}} = atom(P) \cup body(P)$ **then** // *stable model*
  return $A^{\mathbf{T}} \cap atom(P)$
 else
  $\sigma_d := \textsc{Select}(P, \nabla, A)$ // *decision*
  $dl := dl + 1$
  $dlevel(\sigma_d) := dl$
  $A := A \circ \sigma_d$

Observations

- Decision level dl, initially set to 0, is used to count the number of heuristically chosen literals in assignment A
- For a heuristically chosen literal $\sigma_d = \mathbf{T}a$ or $\sigma_d = \mathbf{F}a$, respectively, we require $a \in (atom(P) \cup body(P)) \setminus (A^{\mathbf{T}} \cup A^{\mathbf{F}})$
- For any literal $\sigma \in A$, $dl(\sigma)$ denotes the decision level of σ, viz. the value dl had when σ was assigned
- A conflict is detected from violation of a nogood $\varepsilon \subseteq \Delta_P \cup \nabla$
- A conflict at decision level 0 (where A contains no heuristically chosen literals) indicates non-existence of stable models
- A nogood δ derived by conflict analysis is asserting, that is, some literal is unit-resulting for δ at a decision level $k < dl$
  - After learning δ and backjumping to decision level k, at least one literal is newly derivable by unit propagation
  - No explicit flipping of heuristically chosen literals !

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Example: CDNL-ASP

Consider

$$P = \left\{ \begin{array}{llll} x \leftarrow \sim y & u \leftarrow x, y & v \leftarrow x & w \leftarrow \sim x, \sim y \\ y \leftarrow \sim x & u \leftarrow v & v \leftarrow u, y & \end{array} \right\}$$

dl	σ_d	$\overline{\sigma}$	δ
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		$\mathbf{F}w$	$\{\mathbf{T}w, \mathbf{F}\{\sim x, \sim y\}\} = \delta(w)$
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Outline

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Outline of NogoodPropagation

- Derive deterministic consequences via:
  - Unit propagation on Δ_P and ∇;
  - Unfounded sets $U \subseteq atom(P)$
- Note that U is unfounded if $EB_P(U) \subseteq A^{\mathbf{F}}$
  - Note For any $a \in U$, we have $(\lambda(a, U) \setminus \{\mathbf{T}a\}) \subseteq A$
- An "interesting" unfounded set U satisfies:

$$\emptyset \subset U \subseteq (atom(P) \setminus A^{\mathbf{F}})$$

- Wrt a fixpoint of unit propagation, such an unfounded set contains some loop of P
  - Note Tight programs do not yield "interesting" unfounded sets !
- Given an unfounded set U and some $a \in U$, adding $\lambda(a, U)$ to ∇ triggers a conflict or further derivations by unit propagation
  - Note Add loop nogoods atom by atom to eventually falsify all $a \in U$

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Algorithm 3: NogoodPropagation

Input : A normal program P, a set ∇ of nogoods, and an assignment A.
Output : An extended assignment and set of nogoods.

$U := \emptyset$ // *unfounded set*

loop
  repeat
    if $\delta \subseteq A$ **for some** $\delta \in \Delta_P \cup \nabla$ **then return** (A, ∇) // *conflict*
    $\Sigma := \{\delta \in \Delta_P \cup \nabla \mid \delta \setminus A = \{\overline{\sigma}\}, \sigma \notin A\}$ // *unit-resulting nogoods*
    if $\Sigma \neq \emptyset$ **then let** $\overline{\sigma} \in \delta \setminus A$ **for some** $\delta \in \Sigma$ **in**
      $dlevel(\sigma) := \max(\{dlevel(\rho) \mid \rho \in \delta \setminus \{\overline{\sigma}\}\} \cup \{0\})$
      $A := A \circ \sigma$
  until $\Sigma = \emptyset$
  if $loop(P) = \emptyset$ **then return** (A, ∇)
  $U := U \setminus A^{\mathbf{F}}$
  if $U = \emptyset$ **then** $U :=$ UnfoundedSet(P, A)
  if $U = \emptyset$ **then return** (A, ∇) // *no unfounded set* $\emptyset \subset U \subseteq atom(P) \setminus A^{\mathbf{F}}$
  let $a \in U$ **in**
    $\nabla := \nabla \cup \{\{\mathbf{T}a\} \cup \{\mathbf{F}B \mid B \in EB_P(U)\}\}$ // *record loop nogood*

Requirements for UnfoundedSet

- Implementations of UnfoundedSet must guarantee the following for a result U
  1. $U \subseteq (atom(P) \setminus A^{\mathbf{F}})$
  2. $EB_P(U) \subseteq A^{\mathbf{F}}$
  3. $U = \emptyset$ iff there is no nonempty unfounded subset of $(atom(P) \setminus A^{\mathbf{F}})$
- Beyond that, there are various alternatives, such as:
  - Calculating the greatest unfounded set
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  - Usually, the latter option is implemented in ASP solvers

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Outline

Outline of ConflictAnalysis

- Conflict analysis is triggered whenever some nogood $\delta \in \Delta_P \cup \nabla$ becomes violated, viz. $\delta \subseteq A$, at a decision level $dl > 0$
  - Note that all but the first literal assigned at dl have been unit-resulting for nogoods $\varepsilon \in \Delta_P \cup \nabla$
  - If $\sigma \in \delta$ has been unit-resulting for ε, we obtain a new violated nogood by resolving δ and ε as follows:

$$(\delta \setminus \{\sigma\}) \cup (\varepsilon \setminus \{\overline{\sigma}\})$$

- Resolution is directed by resolving first over the literal $\sigma \in \delta$ derived last, viz. $(\delta \setminus A[\sigma]) = \{\sigma\}$
  - Iterated resolution progresses in inverse order of assignment
- Iterated resolution stops as soon as it generates a nogood δ containing exactly one literal σ assigned at decision level dl
  - This literal σ is called First Unique Implication Point (First-UIP)
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Algorithm 4: CONFLICTANALYSIS

Input : A non-empty violated nogood δ, a normal program P, a set ∇ of nogoods, and an assignment A.

Output : A derived nogood and a decision level.

```
loop
    let σ ∈ δ such that δ \ A[σ] = {σ} in
        k := max({dlevel(ρ) | ρ ∈ δ \ {σ}} ∪ {0})
        if k = dlevel(σ) then
            let ε ∈ Δ_P ∪ ∇ such that ε \ A[σ] = {σ̄} in
                δ := (δ \ {σ}) ∪ (ε \ {σ̄})                // resolution
        else return (δ, k)
```

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2	$\mathbf{F}\{\sim x, \sim y\}$			
		$\mathbf{F}w$	$\{\mathbf{T}w, \mathbf{F}\{\sim x, \sim y\}\} = \delta(w)$	
3	$\mathbf{F}\{\sim y\}$			
		$\mathbf{F}x$	$\{\mathbf{T}x, \mathbf{F}\{\sim y\}\} = \delta(x)$	
		$\mathbf{F}\{x\}$	$\{\mathbf{T}\{x\}, \mathbf{F}x\} \in \Delta(\{x\})$	$\{\mathbf{T}u, \mathbf{F}x\}$
		$\mathbf{F}\{x, y\}$	$\{\mathbf{T}\{x, y\}, \mathbf{F}x\} \in \Delta(\{x, y\})$	$\{\mathbf{T}u, \mathbf{F}x, \mathbf{F}\{x\}\}$
		$\mathbf{T}\{\sim x\}$	$\{\mathbf{F}\{\sim x\}, \mathbf{F}x\} = \delta(\{\sim x\})$	
		$\mathbf{T}y$	$\{\mathbf{F}\{\sim y\}, \mathbf{F}y\} = \delta(\{\sim y\})$	
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		$\mathbf{T}v$	$\{\mathbf{F}v, \mathbf{T}\{u, y\}\} \in \Delta(v)$	
			$\{\mathbf{T}u, \mathbf{F}\{x\}, \mathbf{F}\{x, y\}\} = \lambda(u, \{u, v\})$	✘

Potassco

Example: ConflictAnalysis

Consider

$$P = \left\{ \begin{array}{llll} x \leftarrow \sim y & u \leftarrow x, y & v \leftarrow x & w \leftarrow \sim x, \sim y \\ y \leftarrow \sim x & u \leftarrow v & v \leftarrow u, y & \end{array} \right\}$$

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Remarks

- There always is a First-UIP at which conflict analysis terminates
  - In the worst, resolution stops at the heuristically chosen literal assigned at decision level *dl*
- The nogood δ containing First-UIP σ is violated by A, viz. $\delta \subseteq A$
- We have $k = max(\{dl(\rho) \mid \rho \in \delta \setminus \{\sigma\}\} \cup \{0\}) < dl$
  - After recording δ in ∇ and backjumping to decision level k, $\overline{\sigma}$ is unit-resulting for δ !
  - Such a nogood δ is called asserting
- Asserting nogoods direct conflict-driven search into a different region of the search space than traversed before, without explicitly flipping any heuristically chosen literal !

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Systems: Overview

Outline

potassco.sourceforge.net

Potassco, the Potsdam Answer Set Solving Collection,
bundles tools for ASP developed at the University of Potsdam,
for instance:

- Grounder gringo, lingo, pyngo
- Solver clasp, {a,h,un}clasp, claspD, claspfolio, claspar, aspeed
- Grounder+Solver Clingo, iClingo, {ros}oClingo, Clingcon
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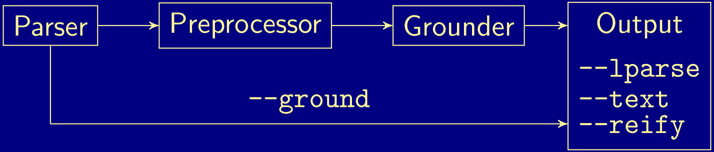
Outline

Potassco

gringo

- Accepts safe programs with aggregates
- Tolerates unrestricted use of function symbols
  (as long as it yields a finite ground instantiation :)
- Expressive power of a Turing machine

- Basic architecture of *gringo*:



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clasp

- *clasp* is a native ASP solver combining conflict-driven search with sophisticated reasoning techniques:
  - advanced preprocessing, including equivalence reasoning
  - lookback-based decision heuristics
  - restart policies
  - nogood deletion
  - progress saving
  - dedicated data structures for binary and ternary nogoods
  - lazy data structures (watched literals) for long nogoods
  - dedicated data structures for cardinality and weight constraints
  - lazy unfounded set checking based on "source pointers"
  - tight integration of unit propagation and unfounded set checking
  - various reasoning modes
  - parallel search
  - ...

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Reasoning modes of *clasp*

- Beyond deciding (stable) model existence, *clasp* allows for:
  - Optimization
  - Enumeration (without solution recording)
  - Projective enumeration (without solution recording)
  - Intersection and Union (linear solution computation)
  - and combinations thereof

- *clasp* allows for
  - ASP solving (*smodels* format)
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Parallel search in *clasp*

- *clasp*
  - pursues a coarse-grained, task-parallel approach to parallel search via shared memory multi-threading
    - up to 64 configurable (non-hierarchic) threads
  - allows for parallel solving via search space splitting and/or competing strategies
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Potassco

Sequential CDCL-style solving

loop

> *propagate* // deterministically assign literals
>
> **if** no conflict **then**
>
> > **if** all variables assigned **then return** solution
> > **else** *decide* // non-deterministically assign some literal
>
> **else**
>
> > **if** top-level conflict **then return** unsatisfiable
> > **else**
> >
> > > *analyze* // analyze conflict and add conflict constraint
> > > *backjump* // unassign literals until conflict constraint is unit

Parallel CDCL-style solving in *clasp*

while work available
  while no (result) message to send
    communicate // exchange information with other solver
    propagate // deterministically assign literals
    if no conflict **then**
      if all variables assigned **then send** solution
      else *decide* // non-deterministically assign some literal
    else
      if root-level conflict **then send** unsatisfiable
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      else
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  communicate // exchange results (and receive work)

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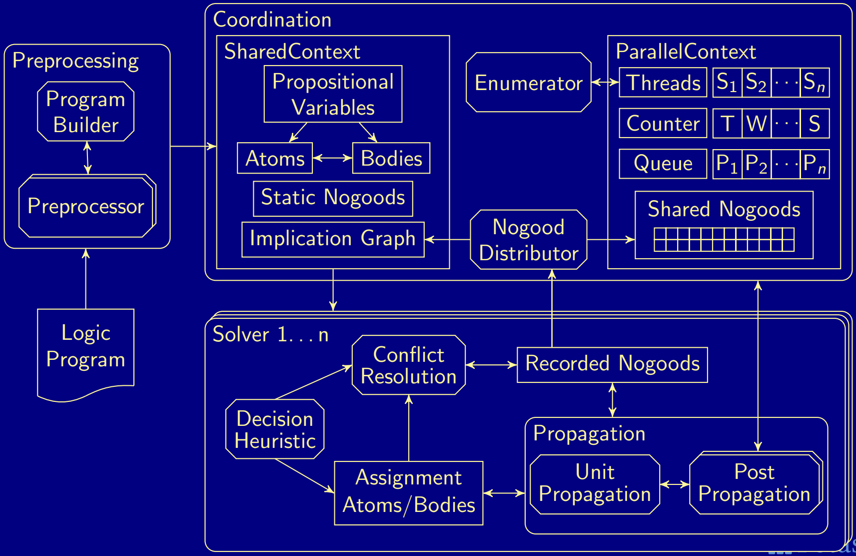
Multi-threaded architecture of *clasp*



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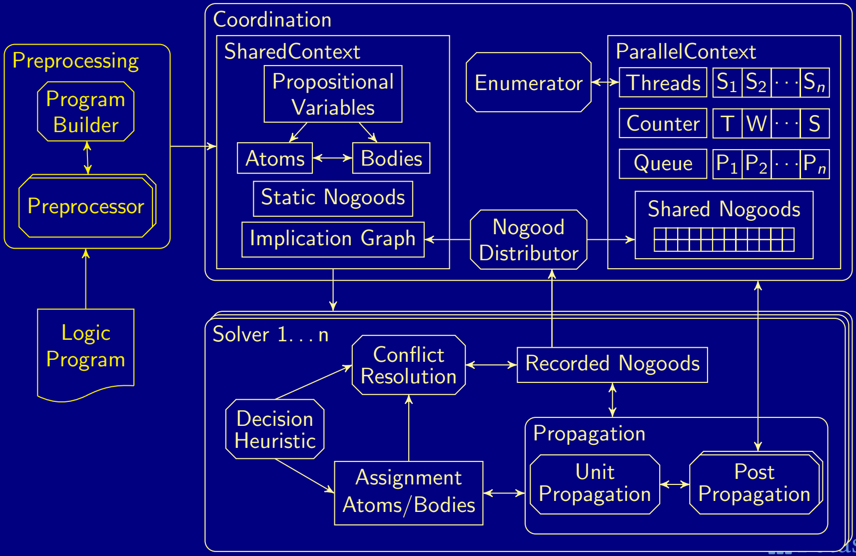

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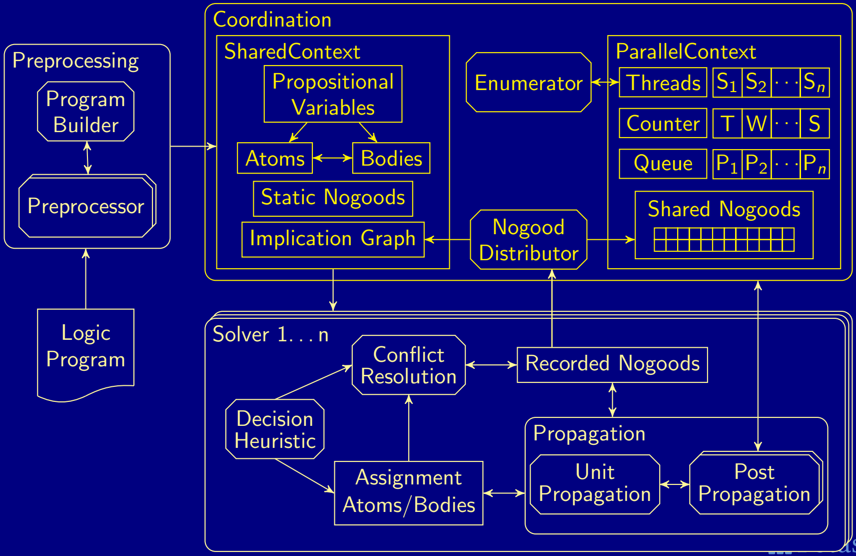

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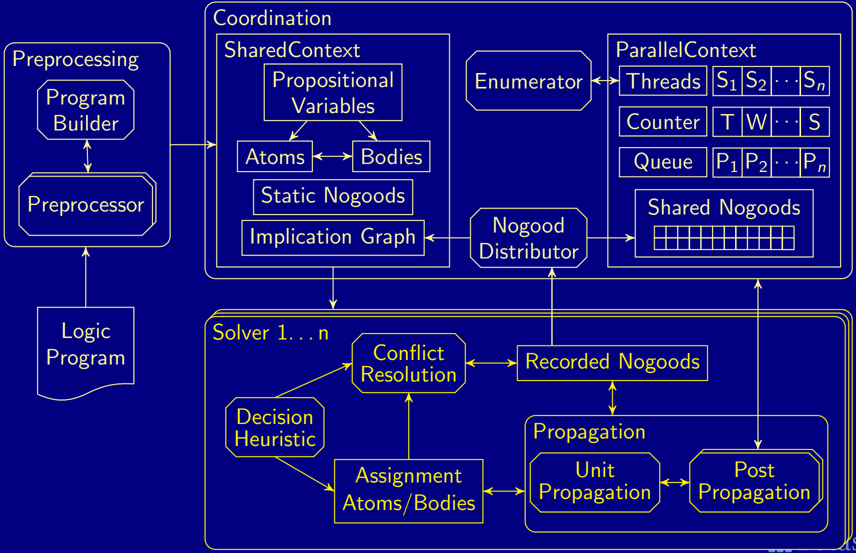


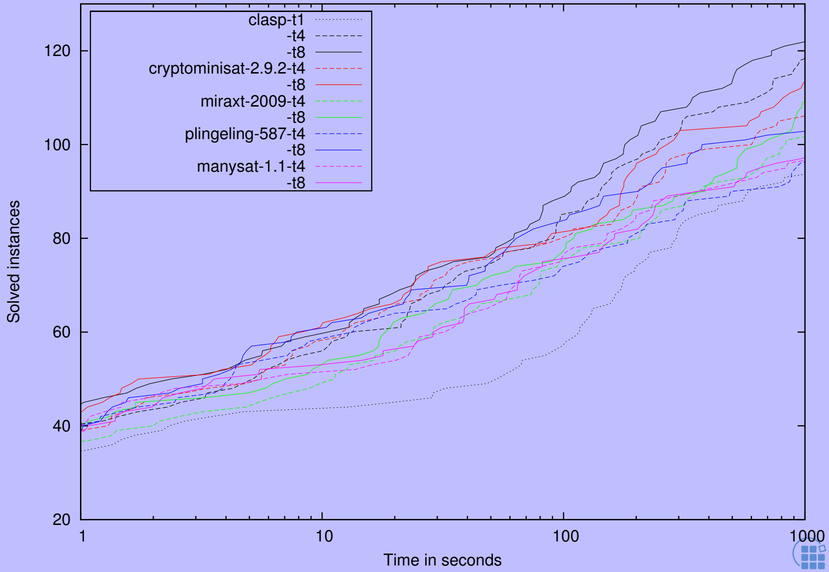
clasp in context

- Compare *clasp* (2.0.5) to the multi-threaded SAT solvers
  - *cryptominisat* (2.9.2)
  - *manysat* (1.1)
  - *miraxt* (2009)
  - *plingeling* (587f)

  all run with four and eight threads in their default settings

- 160/300 benchmarks from crafted category at SAT'11
  - all solvable by *ppfolio* in 1000 seconds
  - crafted SAT benchmarks are closest to ASP benchmarks

clasp in context



Using *clasp*

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--help[=<n>],-h          : Print {1=basic|2=more|3=full} help and exit

--parallel-mode,-t <arg>: Run parallel search with given number of threads
    <arg>: <n {1..64}>[,<mode {compete|split}>]
      <n>   : Number of threads to use in search
      <mode>: Run competition or splitting based search [compete]

--configuration=<arg>    : Configure default configuration [frumpy]
    <arg>: {frumpy|jumpy|handy|crafty|trendy|chatty}
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      handy : Use defaults geared towards large problems
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Comparing configurations

on `queensA.lp`

n	frumpy	jumpy	handy	crafty	trendy	chatty
50	0.063	0.023	3.416	0.030	1.805	0.061
100	20.364	0.099	7.891	0.136	7.321	0.121
150	60.000	0.212	14.522	0.271	19.883	0.347
200	60.000	0.415	15.026	0.667	32.476	0.753
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(times in seconds, cut-off at 60 seconds)

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default

(times in seconds, cut-off at 60 seconds)

Comparing configurations

on `queensA.lp`

n	frumpy	jumpy	handy	crafty	trendy	chatty
50	0.063	0.023	3.416	0.030	1.805	0.061
100	20.364	0.099	7.891	0.136	7.321	0.121
150	60.000	0.212	14.522	0.271	19.883	0.347
200	60.000	0.415	15.026	0.667	32.476	0.753
500	60.000	3.199	60.000	7.471	60.000	6.104

(times in seconds, cut-off at 60 seconds)

Outline

clasp's default portfolio for parallel solving

via `clasp --print-portfolio`

```
[CRAFTY]: --heuristic=vsids --restarts=x,128,1.5 --deletion=3,75,10.0 --del-init-r=1000,9000 --del-grow=1.1,20.
[TRENDY]: --heuristic=vsids --restarts=d,100,0.7 --deletion=3,50 --del-init=500,19500 --del-grow=1.1,20.0,x,100
[FRUMPY]: --heuristic=berkmin --restarts=x,100,1.5 --deletion=1,75 --del-init-r=200,40000 --del-max=400000 --de
[JUMPY]:  --heuristic=vsids --restarts=l,100 --del-init-r=1000,20000 --del-algo=basic,2 --deletion=3,75 --del-g
[STRONG]: --heuristic=berkmin --restarts=x,100,1.5 --deletion=1,75 --del-init-r=200,40000 --del-max=400000 --de
[HANDY]:  --heuristic=vsids --restarts=d,100,0.7 --deletion=2,50,20.0 --del-max=200000 --del-algo=sort,2 --del-
[S2]: --heuristic=vsids --reverse-arcs=1 --otfs=1 --local-restarts --save-progress=0 --contraction=250 --counte
[S4]: --heuristic=vsids --restarts=l,256 --counter-restart=3 --strengthen=recursive --update-lbd --del-glue=2 -
[SLOW]:  --heuristic=berkmin --berk-max=512 --restarts=f,16000 --lookahead=atom,50
[VMTF]:  --heuristic=vmtf --str=no --contr=0 --restarts=x,100,1.3 --del-init-r=800,9200
[SIMPLE]:  --heuristic=vsids  --strengthen=recursive --restarts=x,100,1.5,15 --contraction=0
[LUBY-SP]: --heuristic=vsids --restarts=l,128 --save-p --otfs=1 --init-w=2 --contr=0 --opt-heu=3
[LOCAL-R]: --berk-max=512 --restarts=x,100,1.5,6 --local-restarts --init-w=2 --contr=0
```

- *clasp*'s portfolio is fully customizable
- configurations are assigned in a round-robin fashion to threads during parallel solving
- chatty uses four threads with CRAFTY, TRENDY, FRUMPY, and JUMPY

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Outline

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hclasp

- *hclasp* allows for incorporating domain-specific heuristics into ASP solving
  - input language for expressing domain-specific heuristics
  - solving capacities for integrating domain-specific heuristics
- Example

```
_heuristics(occ(A,T),factor,T) :- action(A), time(T).
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Basic CDCL decision algorithm

loop

propagate // compute deterministic consequences

if no conflict **then**

if all variables assigned **then return** variable assignment

else decide // non-deterministically assign some literal

else

if top-level conflict **then return** unsatisfiable

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analyze // analyze conflict and add a conflict constraint

backjump // undo assignments until conflict constraint is unit

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Inside *decide*

- Heuristic functions

$$h : \mathcal{A} \to [0, +\infty) \quad \text{and} \quad s : \mathcal{A} \to \{\mathbf{T}, \mathbf{F}\}$$

- Algorithmic scheme

1. $h(a) := \alpha \times h(a) + \beta(a)$ for each $a \in \mathcal{A}$
2. $U := \mathcal{A} \setminus (A^{\mathbf{T}} \cup A^{\mathbf{F}})$
3. $C := argmax_{a \in U} h(a)$
4. $a := \tau(C)$
5. $A := A \cup \{a \mapsto s(a)\}$

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Potassco

Heuristic language elements

- Heuristic predicate `_heuristic`

- Heuristic modifiers (atom, *a*, and integer, *v*)

  `init` for initializing the heuristic value of *a* with *v*
  `factor` for amplifying the heuristic value of *a* by factor *v*
  `level` for ranking all atoms; the rank of *a* is *v*
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```
_heuristic(occurs(mv,5),factor,5) :- action(mv), time(5).
```

Simple STRIPS planner

```
time(1..t).

holds(P,0) :- init(P).

1 { occurs(A,T) : action(A) } 1 :- time(T).
 :- occurs(A,T), pre(A,F), not holds(F,T-1).

holds(F,T) :- holds(F,T-1), not nolds(F,T), time(T).
holds(F,T) :- occurs(A,T), add(A,F).
nolds(F,T) :- occurs(A,T), del(A,F).

 :- query(F), not holds(F,t).
```

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holds(F,T) :- occurs(A,T), add(A,F).
nolds(F,T) :- occurs(A,T), del(A,F).

 :- query(F), not holds(F,t).

_heuristic(occurs(A,T),factor,2) :- action(A), time(T).
```

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_heuristic(occurs(A,T),level,1) :- action(A), time(T).
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 :- query(F), not holds(F,t).

_heuristic(A,level,V) :- _heuristic(A,true, V).
_heuristic(A,sign, 1) :- _heuristic(A,true, V).
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_heuristic(A,sign,-1) :- _heuristic(A,false,V).
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nolds(F,T) :- occurs(A,T), del(A,F).

 :- query(F), not holds(F,t).

_heuristic(holds(F,T-1),true, t-T+1) :- holds(F,T).
_heuristic(holds(F,T-1),false,t-T+1) :-
                fluent(F), time(T), not holds(F,T).
```

Heuristic modifications to functions h and s

- $\nu(V_{a,m}(A))$ — *"value for modifier m on atom a wrt assignment A"*
- `init` and

$$d_0(a) = \quad \nu(V_{a,\texttt{init}}(A_0)) + h_0(a)$$

$$d_i(a) = \begin{cases} \nu(V_{a,\texttt{factor}}(A_i)) \times h_i(a) & \text{if } V_{a,\texttt{factor}}(A_i) \neq \emptyset \\ h_i(a) & \text{otherwise} \end{cases}$$

- `sign`

$$t_i(a) = \begin{cases} \mathbf{T} & \text{if } \nu(V_{a,\texttt{sign}}(A_i)) > 0 \\ \mathbf{F} & \text{if } \nu(V_{a,\texttt{sign}}(A_i)) < 0 \\ s_i(a) & \text{otherwise} \end{cases}$$

- `level` $\quad \ell_{A_i}(\mathcal{A}') = argmax_{a \in \mathcal{A}'} \nu(V_{a,\texttt{level}}(A_i)) \qquad \mathcal{A}' \subseteq \mathcal{A}$

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Potassco

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Inside *decide*, heuristically modified

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3. $C := argmax_{a \in U} d(a)$
4. $a := \tau(C)$
5. $A := A \cup \{a \mapsto t(a)\}$

Inside *decide*, heuristically modified

0. $h(a) := d(a)$ for each $a \in \mathcal{A}$
1. $h(a) := \alpha \times h(a) + \beta(a)$ for each $a \in \mathcal{A}$
2. $U := \ell_A(\mathcal{A} \setminus (A^{\mathbf{T}} \cup A^{\mathbf{F}}))$
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Potassco

Selected high scores from systematic experiments

Setting	*Labyrinth*			*Sokoban*			*Hanoi Tower*		
base configuration	9,108*s* (14) 24,545,667			2,844*s* (3) 19,371,267			9,137*s* (11) 41,016,235		
a, `init`, 2	95%	(12)	94%	91%	(1)	84%	85%	(9)	89%
a, `factor`, 4	78%	(8)	30%	120%	(1)	107%	109%	(11)	110%
a, `factor`, 16	78%	(10)	23%	120%	(1)	107%	109%	(11)	110%
a, `level`, 1	90%	(12)	5%	119%	(2)	91%	126%	(15)	120%
f, `init`, 2	103%	(14)	123%	74%	(2)	71%	97%	(10)	109%
f, `factor`, 2	98%	(12)	49%	116%	(3)	134%	55%	(6)	70%
f, `sign`, -1	94%	(13)	89%	105%	(1)	100%	92%	(12)	92%

base configuration versus 38 (static) heuristic modifications (action, a, and fluent, f)

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a, `factor`, 4	78% (8) 30%	120% (1) 107%	109% (11) 110%
a, `factor`, 16	78% (10) 23%	120% (1) 107%	109% (11) 110%
a, `level`, 1	90% (12) 5%	119% (2) 91%	126% (15) 120%
f, `init`, 2	103% (14) 123%	74% (2) 71%	97% (10) 109%
f, `factor`, 2	98% (12) 49%	116% (3) 134%	55% (6) 70%
f, `sign`, -1	94% (13) 89%	105% (1) 100%	92% (12) 92%

base configuration versus 38 (static) heuristic modifications (action, a, and fluent, f)

Abductive problems with optimization

Setting		*Diagnosis*	*Expansion*	*Repair (H)*	*Repair (S)*
base configuration		111.1*s* (115)	161.5*s* (100)	101.3*s* (113)	33.3*s* (27)
`sign,-1`		324.5*s* (407)	7.6*s* (3)	8.4*s* (5)	3.1*s* (0)
`sign,-1`	`factor,2`	310.1*s* (387)	7.4*s* (2)	3.5*s* (0)	3.2*s* (1)
`sign,-1`	`factor,8`	305.9*s* (376)	7.7*s* (2)	3.1*s* (0)	2.9*s* (0)
`sign,-1`	`level,1`	76.1*s* (83)	6.6*s* (2)	0.8*s* (0)	2.2*s* (1)
	`level,1`	77.3*s* (86)	12.9*s* (5)	3.4*s* (0)	2.1*s* (0)

(abducibles subject to optimization)

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(abducibles subject to optimization)

Planning Competition Benchmarks

```
_heuristic(holds(F,T-1),true, t-T+1) :- holds(F,T).
_heuristic(holds(F,T-1),false,t-T+1) :-
                fluent(F), time(T), not holds(F,T).
```

Problem	*base configuration*		_heuristic		*base c. (SAT)*		_heur. (SAT)	
Blocks'00	134.4*s*	(180/61)	9.2*s*	(239/3)	163.2*s*	(59)	2.6*s*	(0)
Elevator'00	3.1*s*	(279/0)	0.0*s*	(279/0)	3.4*s*	(0)	0.0*s*	(0)
Freecell'00	288.7*s*	(147/115)	184.2*s*	(194/74)	226.4*s*	(47)	52.0*s*	(0)
Logistics'00	145.8*s*	(148/61)	115.3*s*	(168/52)	113.9*s*	(23)	15.5*s*	(3)
Depots'02	400.3*s*	(51/184)	297.4*s*	(115/135)	389.0*s*	(64)	61.6*s*	(0)
Driverlog'02	308.3*s*	(108/143)	189.6*s*	(169/92)	245.8*s*	(61)	6.1*s*	(0)
Rovers'02	245.8*s*	(138/112)	165.7*s*	(179/79)	162.9*s*	(41)	5.7*s*	(0)
Satellite'02	398.4*s*	(73/186)	229.9*s*	(155/106)	364.6*s*	(82)	30.8*s*	(0)
Zenotravel'02	350.7*s*	(101/169)	239.0*s*	(154/116)	224.5*s*	(53)	6.3*s*	(0)
Total	252.8*s*	(1225/1031)	158.9*s*	(1652/657)	187.2*s*	(430)	17.1*s*	(3)

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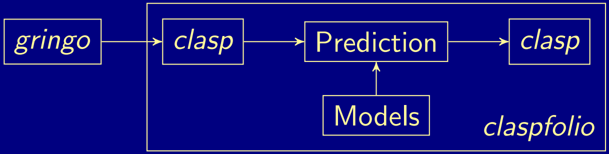
Outline

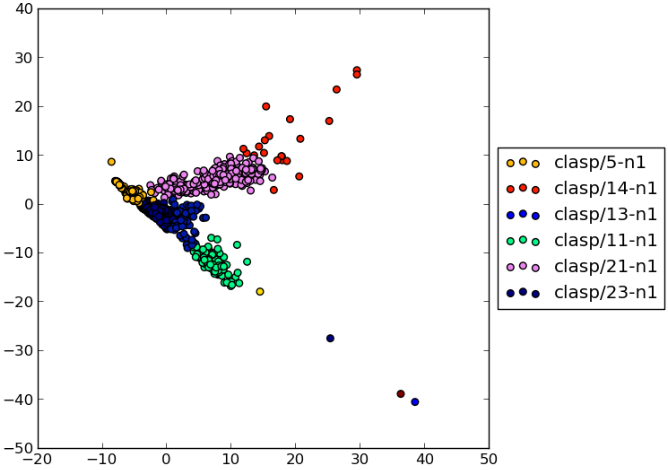
claspfolio

- Automatic selection of some *clasp* configuration among several predefined ones via (learned) classifiers

- Basic architecture of *claspfolio*:



Instance Feature Clusters (after PCA)



Solving with *clasp* (as usual)

```
$ clasp queens500 --quiet

clasp version 2.0.2
Reading from queens500
Solving...
SATISFIABLE

Models      : 1+
Time        : 11.445s (Solving: 10.58s 1st Model: 10.55s Unsat: 0.00s)
CPU Time    : 11.410s
```

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```

Potassco

Solving with *claspfolio*

```
$ claspfolio queens500 --quiet

PRESOLVING
Reading from queens500
Solving...
claspfolio version 1.0.1 (based on clasp version 2.0.2)
Reading from queens500
Solving...
SATISFIABLE

Models       : 1+
Time         : 4.785s (Solving: 3.96s 1st Model: 3.92s Unsat: 0.00s)
CPU Time     : 4.780s
```

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Solving...
claspfolio version 1.0.1 (based on clasp version 2.0.2)
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Feature-extraction with *claspfolio*

```
$ claspfolio --features queens500

PRESOLVING
Reading from queens500
Solving...
UNKNOWN
Features     : 84998,3994,0,250000,1.020,62.594,63.844,21.281,84998, \
 3994,100,250000,1.020,62.594,63.844,21.281,84998,3994,250,250000,  \
 1.020,62.594,63.844,21.281,84998,3994,475,250000,1.020,62.594,     \
 63.844,21.281,757989,757989,0,510983,506992,3990,1,0,127.066,9983, \
 1023958,502993,1994,518971,1,0,0,254994,0,3990,0.100,0.000,99.900, \
 0,270303,812,4,0,812,2223,2223,262,262,2.738,2.738,0.000,812,812,  \
 2270.982,0,0.000

$ claspfolio --list-features

maxLearnt,Constraints,LearntConstraints,FreeVars,Vars/FreeVars, ...
```

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Solving...
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Prediction with *claspfolio*

```
$ claspfolio queens500 --decisionvalues

PRESOLVING
Reading from queens500
Solving...

Portfolio Decision Values:
[1] : 3.437538   [10] : 3.639444   [19] : 3.726391
[2] : 3.501728   [11] : 3.483334   [20] : 3.020325
[3] : 3.784733   [12] : 3.271890   [21] : 3.220219
[4] : 3.672955   [13] : 3.344085   [22] : 3.998709
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Solving with *claspfolio* (slightly verbosely)

```
$ claspfolio queens500 --quiet --autoverbose=1

PRESOLVING
Reading from queens500
Solving...

Chosen configuration: [20]
clasp --configurations=./models/portfolio.txt           \
      --modelpath=./models/                             \
      queens500 --quiet --autoverbose=1                 \
      --heu=VSIDS --sat-pre=20,25,120 --trans-ext=integ

claspfolio version 1.0.1 (based on clasp version 2.0.2)
Reading from queens500
Solving...
SATISFIABLE

Models       : 1+
Time         : 4.783s (Solving: 3.96s 1st Model: 3.93s Unsat: 0.00s)
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Outline

claspD

- *claspD* is a multi-threaded solver for disjunctive logic programs
- aiming at an equitable interplay between "generating" and "testing" solver units
- allowing for a bidirectional dynamic information exchange between solver units for orthogonal tasks

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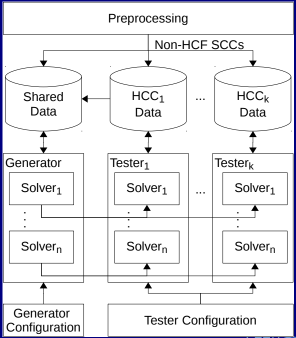
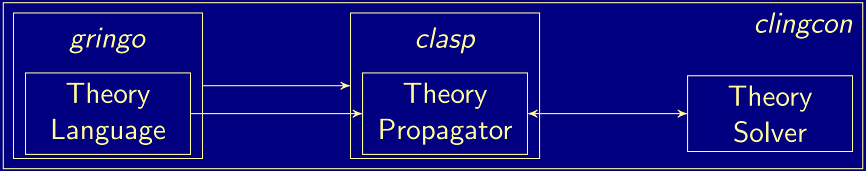


Outline

clingcon

- Hybrid grounding and solving
- Solving in hybrid domains, like Bio-Informatics

- Basic architecture of *clingcon*:



Pouring Water into Buckets on a Scale

```
time(0..t).               $domain(0..500).
bucket(a).                volume(a,0) $== 0.
bucket(b).                volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

          1 $<= amount(B,T) :- pour(B,T), T < t.
amount(B,T) $<= 30          :- pour(B,T), T < t.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
time(0..t).                $domain(0..500).
bucket(a).                 volume(a,0) $== 0.
bucket(b).                 volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

          1 $<= amount(B,T) :- pour(B,T), T < t.
amount(B,T) $<= 30          :- pour(B,T), T < t.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
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bucket(a).               volume(a,0) $== 0.
bucket(b).               volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

          1 $<= amount(B,T) :- pour(B,T), T < t.
amount(B,T) $<= 30          :- pour(B,T), T < t.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
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bucket(a).               volume(a,0) $== 0.
bucket(b).               volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

          1 $<= amount(B,T) :- pour(B,T), T < t.
amount(B,T) $<= 30          :- pour(B,T), T < t.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
time(0..t).            $domain(0..500).
bucket(a).             volume(a,0) $== 0.
bucket(b).             volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

 :- pour(B,T), T < t, not (1 $<= amount(B,T)).
amount(B,T) $<= 30         :- pour(B,T), T < t.
amount(B,T) $==  0         :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Potassco

Pouring Water into Buckets on a Scale

```
time(0..t).             $domain(0..500).
bucket(a).              volume(a,0) $== 0.
bucket(b).              volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

 :- pour(B,T), T < t, 1 $> amount(B,T).
amount(B,T) $<= 30           :- pour(B,T), T < t.
amount(B,T) $==  0           :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
time(0..t).               $domain(0..500).
bucket(a).                volume(a,0) $== 0.
bucket(b).                volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

 :- pour(B,T), T < t, 1 $> amount(B,T).
 :- pour(B,T), T < t, amount(B,T) $> 30.
amount(B,T) $==  0          :- not pour(B,T), bucket(B), time(T), T < t.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
time(0..t).              $domain(0..500).
bucket(a).               volume(a,0) $== 0.
bucket(b).               volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

 :- pour(B,T), T < t, 1 $> amount(B,T).
 :- pour(B,T), T < t, amount(B,T) $> 30.
 :- not pour(B,T), bucket(B), time(T), T < t, amount(B,T) $!=  0.

volume(B,T+1) $== volume(B,T) $+ amount(B,T) :- bucket(B), time(T), T < t.

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Potassco

Pouring Water into Buckets on a Scale

```
time(0..t).            $domain(0..500).
bucket(a).             volume(a,0) $== 0.
bucket(b).             volume(b,0) $== 100.

1 { pour(B,T) : bucket(B) } 1 :- time(T), T < t.

 :- pour(B,T), T < t, 1 $> amount(B,T).
 :- pour(B,T), T < t, amount(B,T) $> 30.
 :- not pour(B,T), bucket(B), time(T), T < t, amount(B,T) $!=  0.

 :- bucket(B), time(T), T < t, volume(B,T+1) $!= volume(B,T)$+amount(B,T).

down(B,T) :- volume(C,T) $< volume(B,T), bucket(B;C), time(T).
  up(B,T) :- not down(B,T), bucket(B), time(T).

 :- up(a,t).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text
```

```
time(0). ... time(4).                                 $domain(0..500).
bucket(a).                                             :- volume(a,0) $!= 0.
bucket(b).                                             :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                     ... 1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                  ...  :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                  ...  :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                 ...  :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                 ...  :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.             ...  :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.             ...  :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)). ...  :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)). ...  :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).          ... down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).          ... down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).          ... down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).          ... down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                         ... up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                         ... up(b,4) :- not down(b,4).

 :- up(a,4).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                                  $domain(0..500).
bucket(a).                                              :- volume(a,0) $!= 0.
bucket(b).                                              :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                      ...  1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                   ...   :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                   ...   :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                  ...   :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                  ...   :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.              ...   :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.              ...   :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)).  ...   :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)).  ...   :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).           ...  down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).           ...  down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).           ...  down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).           ...  down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                          ...  up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                          ...  up(b,4) :- not down(b,4).

 :- up(a,4).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                                  $domain(0..500).
bucket(a).                                              :- volume(a,0) $!= 0.
bucket(b).                                              :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                      ...  1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                   ...   :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                   ...   :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                  ...   :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                  ...   :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.              ...   :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.              ...   :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)).  ...   :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)).  ...   :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).           ...  down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).           ...  down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).           ...  down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).           ...  down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                          ...  up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                          ...  up(b,4) :- not down(b,4).

 :- up(a,4).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                              $domain(0..500).
bucket(a).                                          :- volume(a,0) $!= 0.
bucket(b).                                          :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                  ... 1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).               ...  :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).               ...  :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.              ...  :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.              ...  :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.          ...  :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.          ...  :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)). ...  :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)). ...  :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).       ... down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).       ... down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).       ... down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).       ... down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                      ... up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                      ... up(b,4) :- not down(b,4).

 :- up(a,4).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --text

time(0). ... time(4).                                  $domain(0..500).
bucket(a).                                              :- volume(a,0) $!= 0.
bucket(b).                                              :- volume(b,0) $!= 100.

1 { pour(b,0), pour(a,0) } 1.                      ...  1 { pour(b,3), pour(a,3) } 1.

 :- pour(a,0), 1 $> amount(a,0).                   ...   :- pour(a,3), 1 $> amount(a,3).
 :- pour(b,0), 1 $> amount(b,0).                   ...   :- pour(b,3), 1 $> amount(b,3).

 :- pour(a,0), amount(a,0) $> 30.                  ...   :- pour(a,3), amount(a,3) $> 30.
 :- pour(b,0), amount(b,0) $> 30.                  ...   :- pour(b,3), amount(b,3) $> 30.

 :- not pour(a,0), amount(a,0) $!= 0.              ...   :- not pour(a,3), amount(a,3) $!= 0.
 :- not pour(b,0), amount(b,0) $!= 0.              ...   :- not pour(b,3), amount(b,3) $!= 0.

 :- volume(a,1) $!= (volume(a,0) $+ amount(a,0)).  ...   :- volume(a,4) $!= (volume(a,3) $+ amount(a,3)).
 :- volume(b,1) $!= (volume(b,0) $+ amount(b,0)).  ...   :- volume(b,4) $!= (volume(b,3) $+ amount(b,3)).

down(a,0) :- volume(a,0) $< volume(a,0).           ...  down(a,4) :- volume(a,4) $< volume(a,4).
down(a,0) :- volume(b,0) $< volume(a,0).           ...  down(a,4) :- volume(b,4) $< volume(a,4).
down(b,0) :- volume(a,0) $< volume(b,0).           ...  down(b,4) :- volume(a,4) $< volume(b,4).
down(b,0) :- volume(b,0) $< volume(b,0).           ...  down(b,4) :- volume(b,4) $< volume(b,4).

up(a,0) :- not down(a,0).                          ...  up(a,4) :- not down(a,4).
up(b,0) :- not down(b,0).                          ...  up(b,4) :- not down(b,4).

 :- up(a,4).
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0         1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0         1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0         1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0         1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100       volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100       volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100       volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100       volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100       volume(b,4) $< volume(a,4)

SATISFIABLE

Models       : 1
Time         : 0.000
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0         1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0         1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0         1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0         1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100       volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100       volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100       volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100       volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100       volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0         1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0         1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0         1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0         1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100       volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100       volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100       volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100       volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100       volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1
Time        : 0.000
```

Boolean variables

Potassco

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp 0

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=[11..30]   amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=[11..30]   amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=[11..30]   amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=[11..30]   amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=[11..30]   volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=[41..60]   volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=[71..90]   volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=[101..120] volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models       : 1
Time         : 0.000
```

Non-Boolean variables

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --csp-num-as=1

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=11         amount(b,0)=0         1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=30         amount(b,1)=0         1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=30         amount(b,2)=0         1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=30         amount(b,3)=0         1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0          volume(b,0)=100       volume(a,0) $< volume(b,0)
volume(a,1)=11         volume(b,1)=100       volume(a,1) $< volume(b,1)
volume(a,2)=41         volume(b,2)=100       volume(a,2) $< volume(b,2)
volume(a,3)=71         volume(b,3)=100       volume(a,3) $< volume(b,3)
volume(a,4)=101        volume(b,4)=100       volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1+
Time        : 0.000
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --csp-num-as=1

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=11        amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=30        amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=30        amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=30        amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0         volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=11        volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=41        volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=71        volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=101       volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models       : 1+
Time         : 0.000
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --csp-num-as=1

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=11        amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=30        amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=30        amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=30        amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0         volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=11        volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=41        volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=71        volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=101       volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

Models      : 1+
Time        : 0.000
```

Pouring Water into Buckets on a Scale

```
$ clingcon --const t=4 balance.lp --csp-num-as=1

Answer: 1
pour(a,0)   pour(a,1)   pour(a,2)   pour(a,3)

amount(a,0)=11        amount(b,0)=0        1 $> amount(b,0)   amount(a,0) $!= 0
amount(a,1)=30        amount(b,1)=0        1 $> amount(b,1)   amount(a,1) $!= 0
amount(a,2)=30        amount(b,2)=0        1 $> amount(b,2)   amount(a,2) $!= 0
amount(a,3)=30        amount(b,3)=0        1 $> amount(b,3)   amount(a,3) $!= 0

volume(a,0)=0         volume(b,0)=100      volume(a,0) $< volume(b,0)
volume(a,1)=11        volume(b,1)=100      volume(a,1) $< volume(b,1)
volume(a,2)=41        volume(b,2)=100      volume(a,2) $< volume(b,2)
volume(a,3)=71        volume(b,3)=100      volume(a,3) $< volume(b,3)
volume(a,4)=101       volume(b,4)=100      volume(b,4) $< volume(a,4)

SATISFIABLE

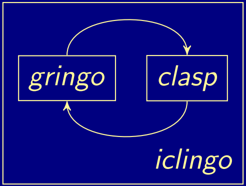
Models      : 1+
Time        : 0.000
```

Outline

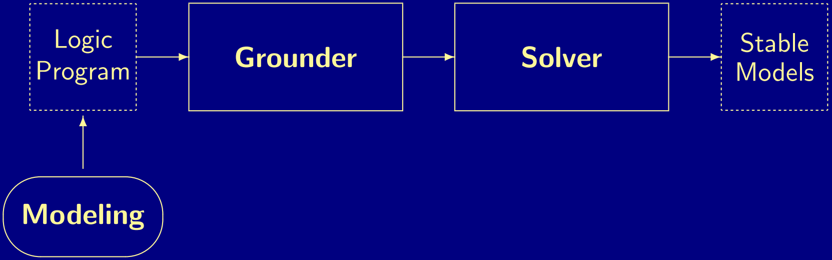
iclingo

- Incremental grounding and solving
- Offline solving in dynamic domains, like Automated Planning

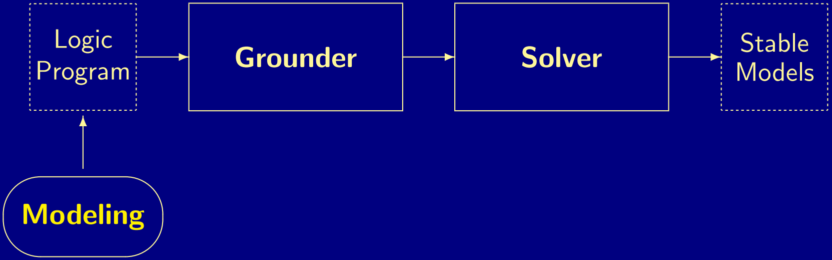
- Basic architecture of *iclingo*:



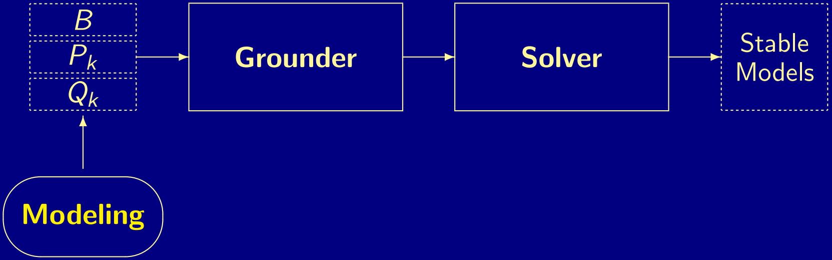
Incremental ASP Solving Process



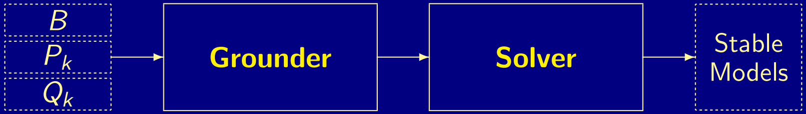
Incremental ASP Solving Process



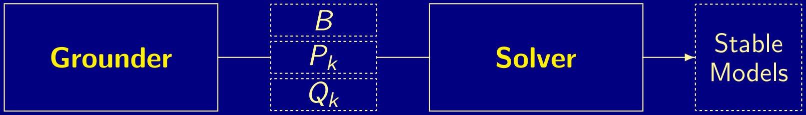
Incremental ASP Solving Process



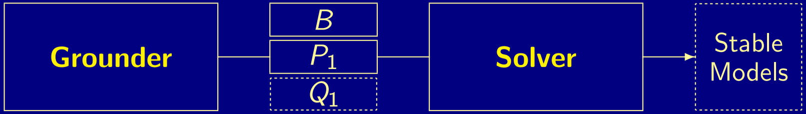
Incremental ASP Solving Process



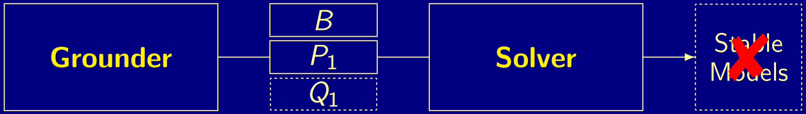
Incremental ASP Solving Process



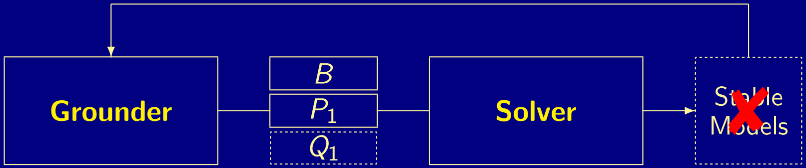
Incremental ASP Solving Process



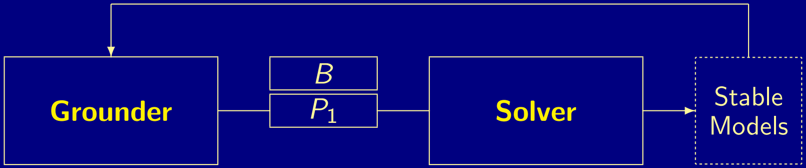
Incremental ASP Solving Process



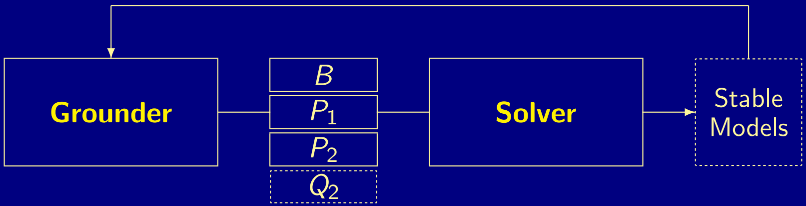
Incremental ASP Solving Process



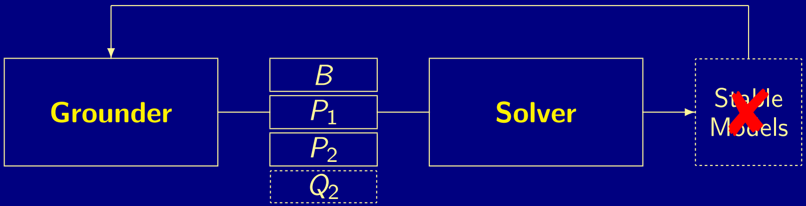
Incremental ASP Solving Process



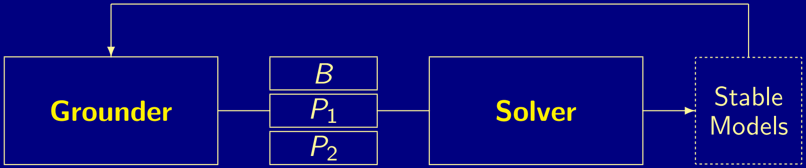
Incremental ASP Solving Process



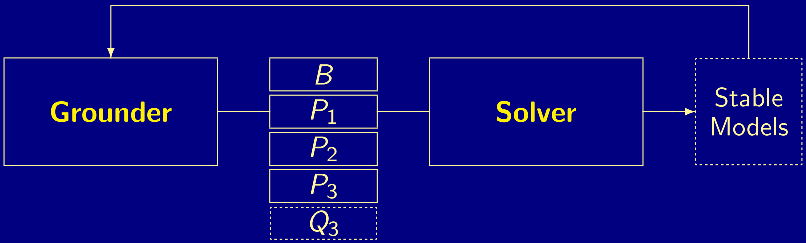
Incremental ASP Solving Process



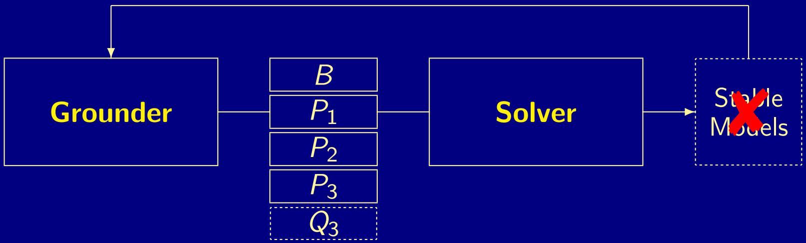
Incremental ASP Solving Process



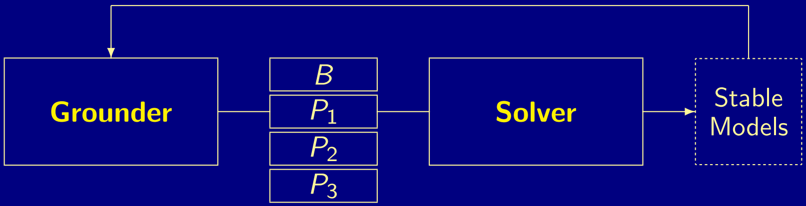
Incremental ASP Solving Process



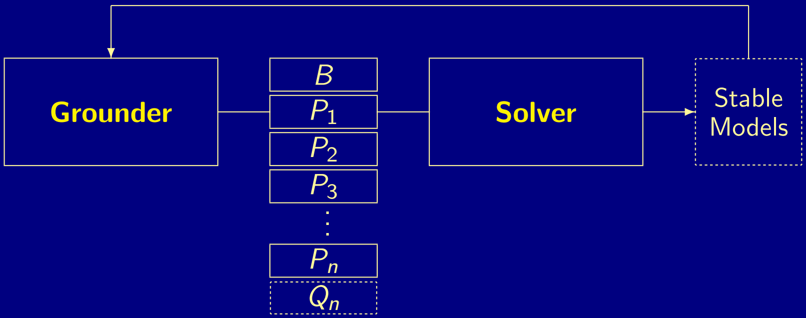
Incremental ASP Solving Process



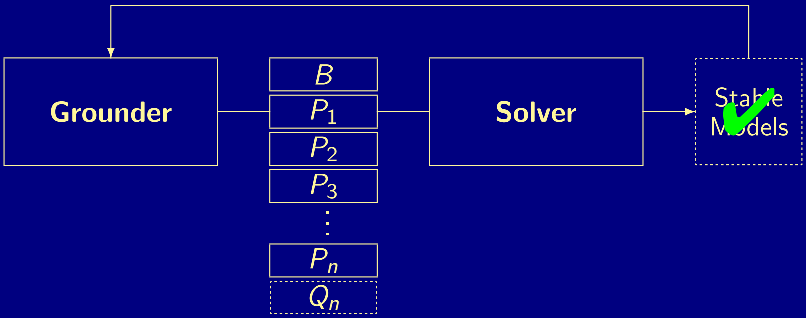
Incremental ASP Solving Process



Incremental ASP Solving Process



Incremental ASP Solving Process



Simplistic STRIPS Planning

```
#base.
fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).     query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

#cumulative t.
1 { occ(A,t) : action(A) } 1.
 :- occ(A,t), pre(A,F), not holds(F,t-1).

holds(F,t) :- holds(F,t-1), not nolds(F,t).
holds(F,t) :- occ(A,t), add(A,F).
nolds(F,t) :- occ(A,t), del(A,F).

#volatile t.
 :- query(F), not holds(F,t).

#hide. #show occ/2.
```

Simplistic STRIPS Planning

```
#base.
fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).      query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

#cumulative t.
1 { occ(A,t) : action(A) } 1.
 :- occ(A,t), pre(A,F), not holds(F,t-1).

holds(F,t) :- holds(F,t-1), not nolds(F,t).
holds(F,t) :- occ(A,t), add(A,F).
nolds(F,t) :- occ(A,t), del(A,F).

#volatile t.
 :- query(F), not holds(F,t).

#hide. #show occ/2.
```

Simplistic STRIPS Planning

```
#base.
fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).     query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

#cumulative t.
1 { occ(A,t) : action(A) } 1.
 :- occ(A,t), pre(A,F), not holds(F,t-1).

holds(F,t) :- holds(F,t-1), not nolds(F,t).
holds(F,t) :- occ(A,t), add(A,F).
nolds(F,t) :- occ(A,t), del(A,F).

#volatile t.
 :- query(F), not holds(F,t).

#hide. #show occ/2.
```

Simplistic STRIPS Planning

```
#base.
fluent(p).     action(a).     action(b).      init(p).
fluent(q).        pre(a,p).      pre(b,q).
fluent(r).        add(a,q).      add(b,r).     query(r).
                  del(a,p).      del(b,q).

holds(P,0) :- init(P).

#cumulative t.
1 { occ(A,t) : action(A) } 1.
 :- occ(A,t), pre(A,F), not holds(F,t-1).

holds(F,t) :- holds(F,t-1), not nolds(F,t).
holds(F,t) :- occ(A,t), add(A,F).
nolds(F,t) :- occ(A,t), del(A,F).

#volatile t.
 :- query(F), not holds(F,t).

#hide. #show occ/2.
```

Simplistic STRIPS Planning

```
$ iclingo iplanning.lp

Answer: 1
occ(a,1) occ(b,2)
SATISFIABLE

Models      : 1
Total Steps : 2
Time        : 0.000
```

Simplistic STRIPS Planning

```
$ iclingo iplanning.lp

Answer: 1
occ(a,1) occ(b,2)
SATISFIABLE

Models      : 1
Total Steps : 2
Time        : 0.000
```

Simplistic STRIPS Planning

```
$ iclingo iplanning.lp --istats

=============== step 1 ===============

Models   : 0
Time     : 0.000 (g: 0.000, p: 0.000, s: 0.000)
Rules    : 27
Choices  : 0
Conflicts: 0
=============== step 2 ===============
Answer: 1
occ(a,1) occ(b,2)

Models   : 1
Time     : 0.000 (g: 0.000, p: 0.000, s: 0.000)
Rules    : 16
Choices  : 0
Conflicts: 0
=============== Summary ===============
SATISFIABLE

Models      : 1
Total Steps : 2
Time        : 0.000
```

Simplistic STRIPS Planning

```
$ iclingo iplanning.lp --istats

=============== step 1 ===============

Models   : 0
Time     : 0.000 (g: 0.000, p: 0.000, s: 0.000)
Rules    : 27
Choices  : 0
Conflicts: 0
=============== step 2 ===============
Answer: 1
occ(a,1) occ(b,2)

Models   : 1
Time     : 0.000 (g: 0.000, p: 0.000, s: 0.000)
Rules    : 16
Choices  : 0
Conflicts: 0
=============== Summary ===============
SATISFIABLE

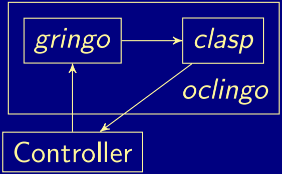
Models      : 1
Total Steps : 2
Time        : 0.000
```

Outline

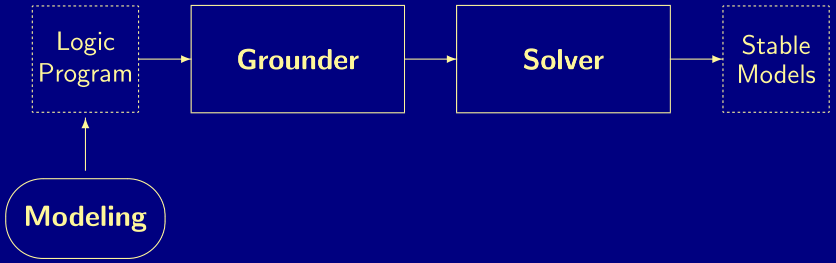
oclingo

- Reactive grounding and solving
- Online solving in dynamic domains, like Robotics

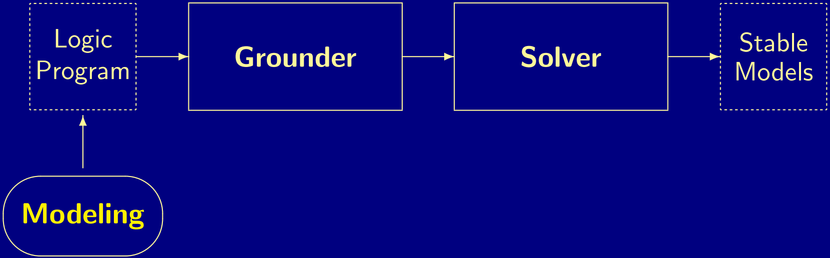
- Basic architecture of *oclingo*:



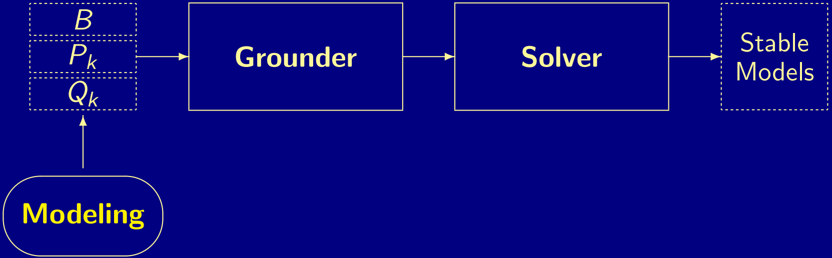
Reactive ASP Solving Process



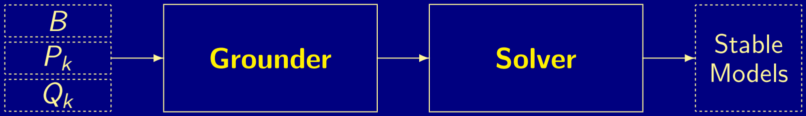
Reactive ASP Solving Process



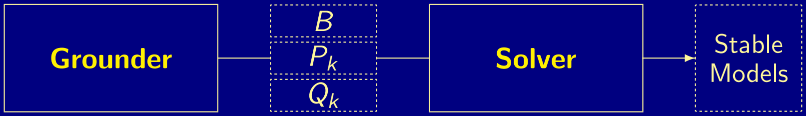
Reactive ASP Solving Process



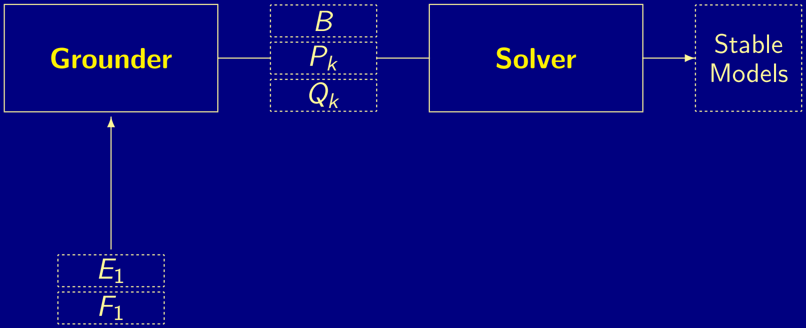
Reactive ASP Solving Process



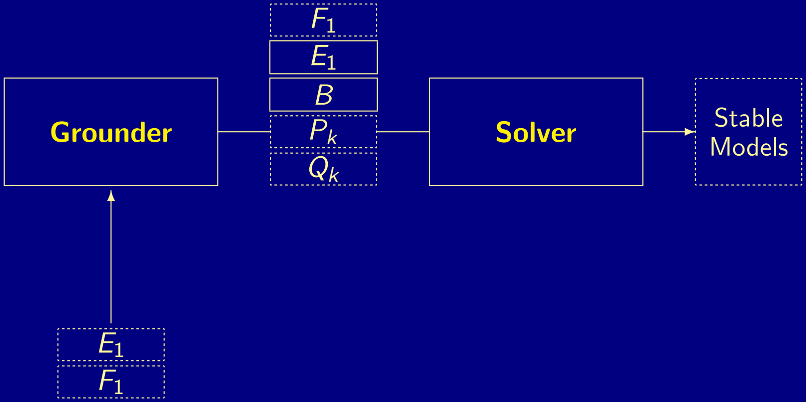
Reactive ASP Solving Process



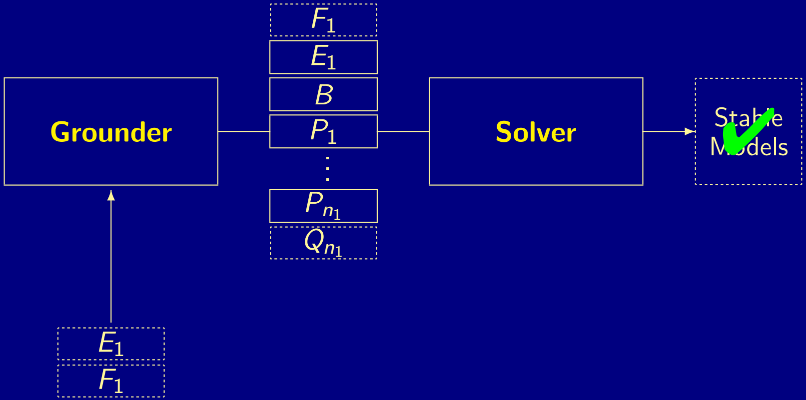
Reactive ASP Solving Process



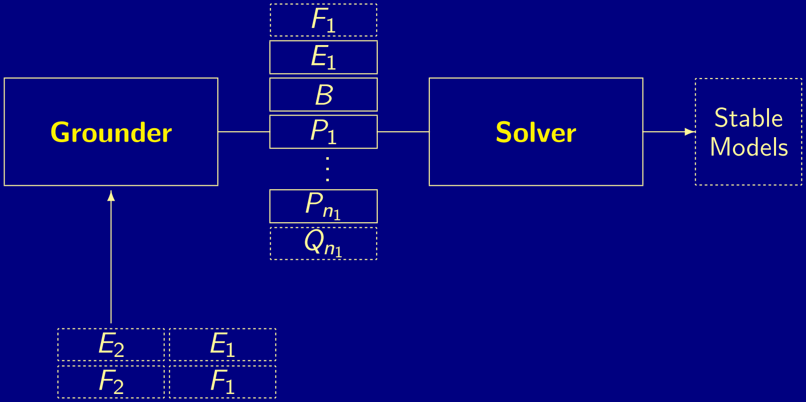
Reactive ASP Solving Process



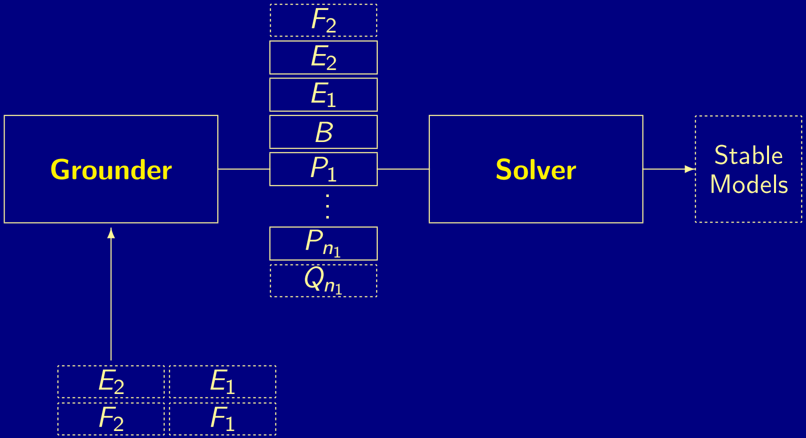
Reactive ASP Solving Process



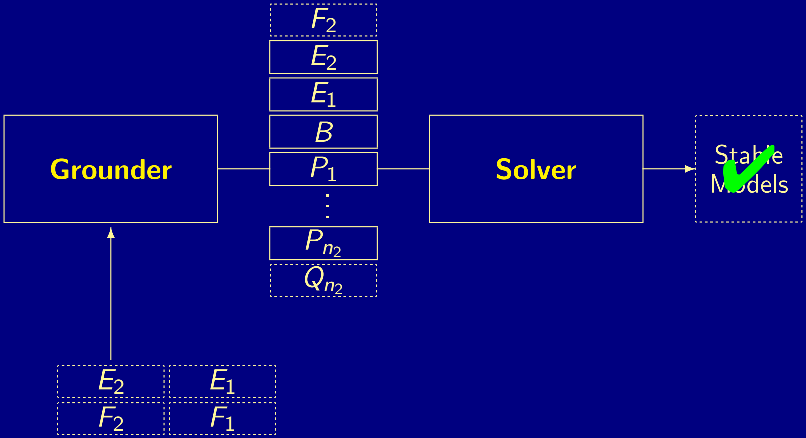
Reactive ASP Solving Process



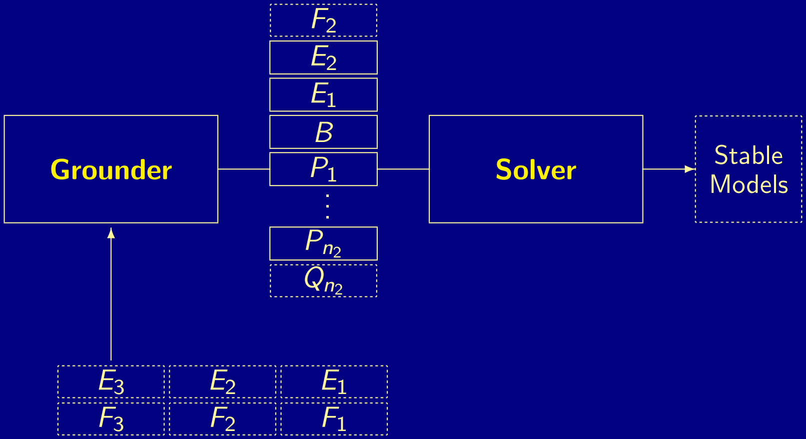
Reactive ASP Solving Process



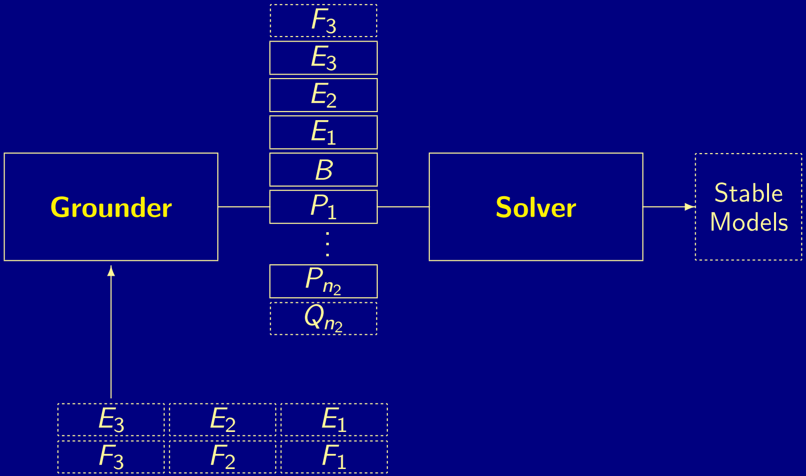
Reactive ASP Solving Process



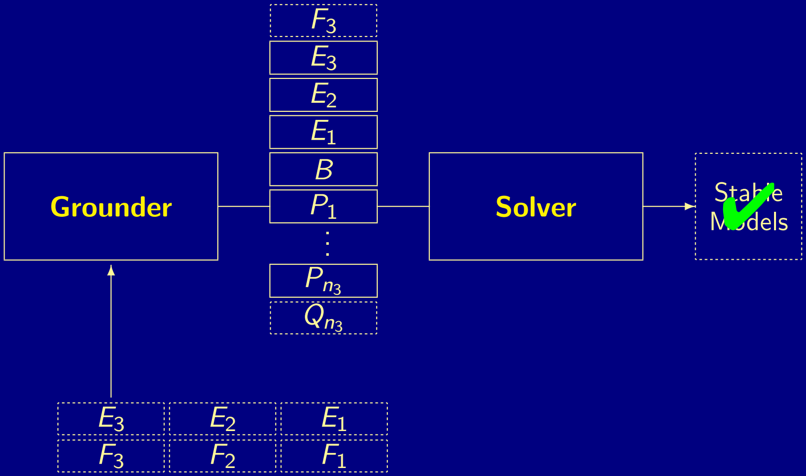
Reactive ASP Solving Process



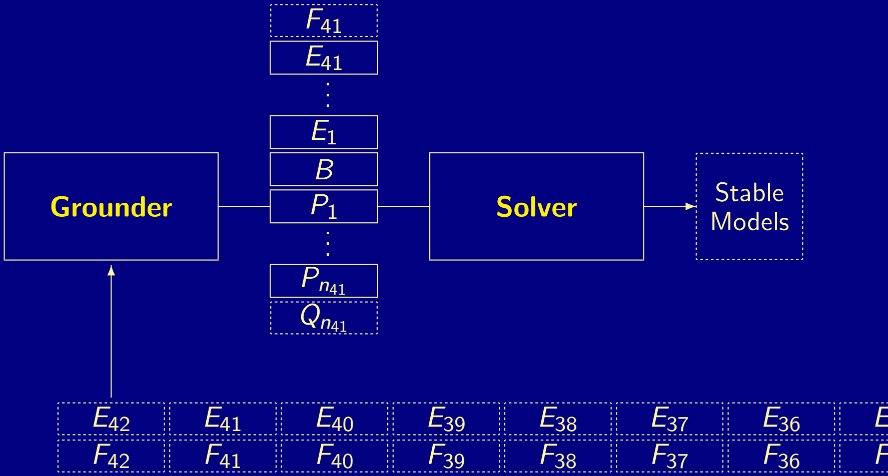
Reactive ASP Solving Process



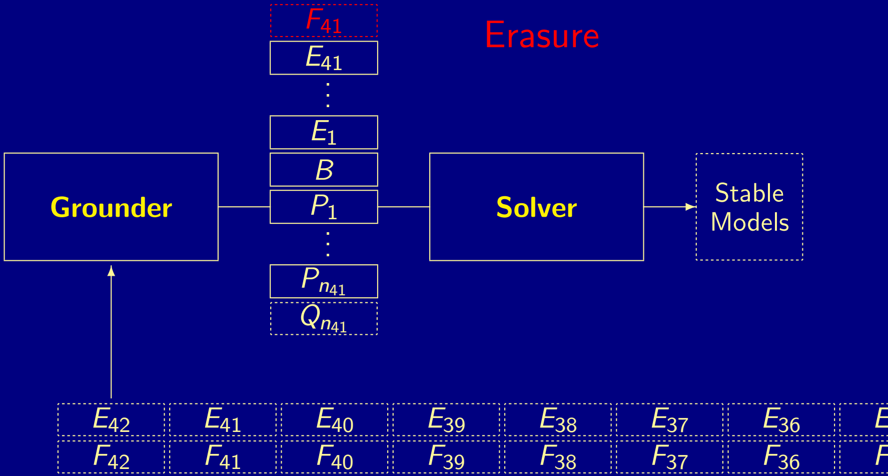
Reactive ASP Solving Process



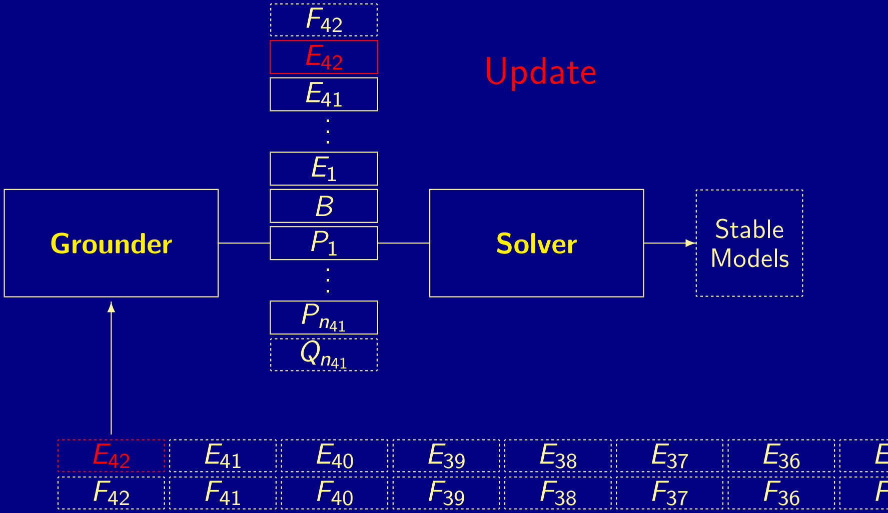
Reactive ASP Solving Process



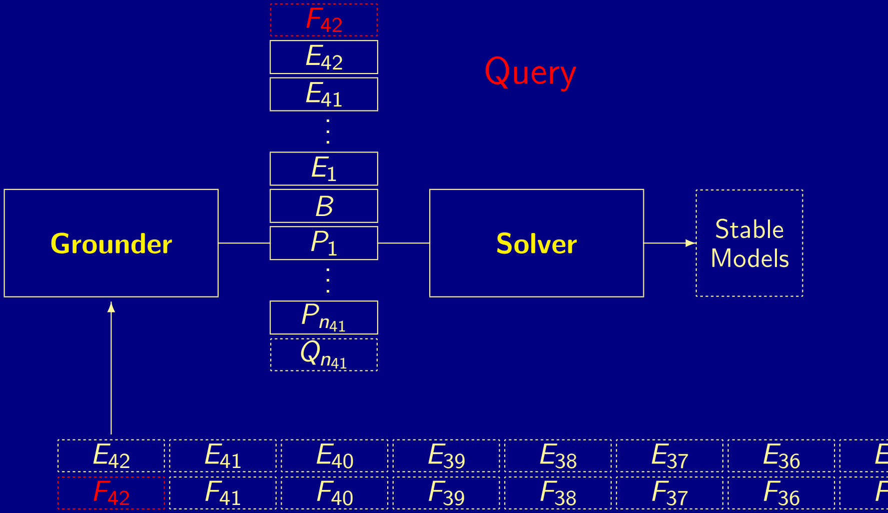
Reactive ASP Solving Process



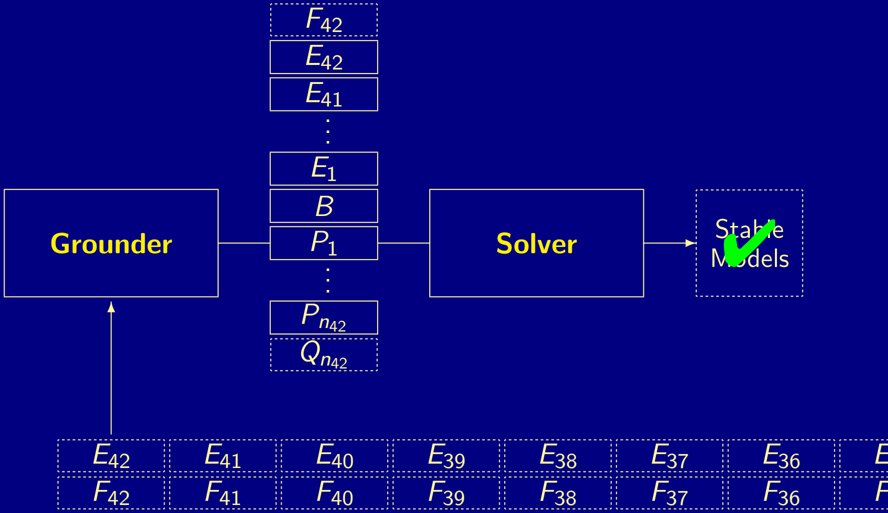
Reactive ASP Solving Process



Reactive ASP Solving Process



Reactive ASP Solving Process



Potassco

Elevator Control

```
#base.
floor(1..3).
atFloor(1,0).

#cumulative t.
#external request(F,t) : floor(F).
1 { atFloor(F-1;F+1,t) } 1 :- atFloor(F,t-1), floor(F).
:- atFloor(F,t), not floor(F).
requested(F,t) :- request(F,t),     floor(F), not atFloor(F,t).
requested(F,t) :- requested(F,t-1), floor(F), not atFloor(F,t).
goal(t) :- not requested(F,t) : floor(F).

#volatile t.
:- not goal(t).
```

Potassco

Pushing a button

- oClingo acts as a server listening on a port waiting for client requests
- To issue such requests, a separate controller program sends online progressions using network sockets
- For instance,

```
#step 1.
request(3,1).
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- This process terminates when the client sends

```
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Outline

clavis

- Analysis and visualization toolchain for clasp
- *clavis*
  - Event logger integrated in clasp
  - Records CDCL events like propagation, conflicts, restarts, ...
  - Generated logfiles readable with different backends
  - Easily configurable
  - Applicable to clasp variants like hclasp
- *insight*
  - Visualization backend for clavis
  - Combines information about problem structure and solving process
  - Networks for structural and aggregated information
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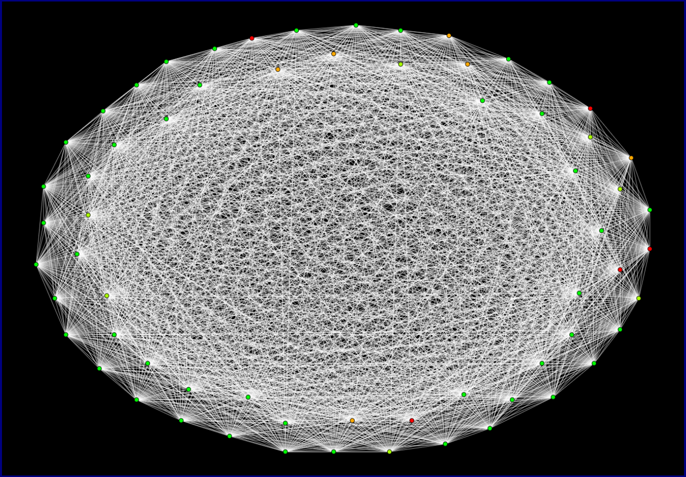
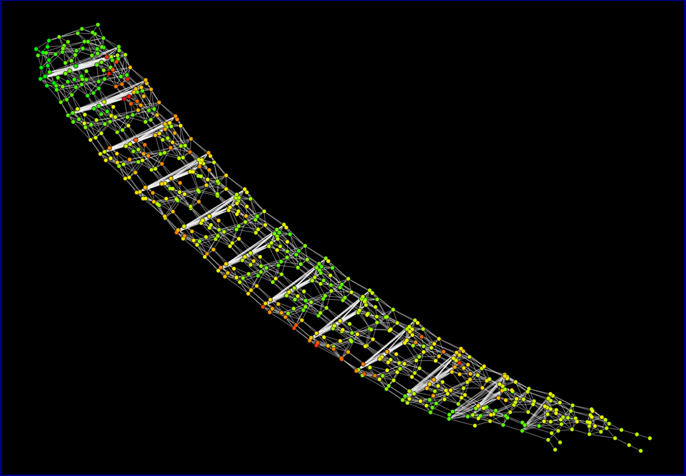
Visualization Examples

8-Queens: program interaction graph



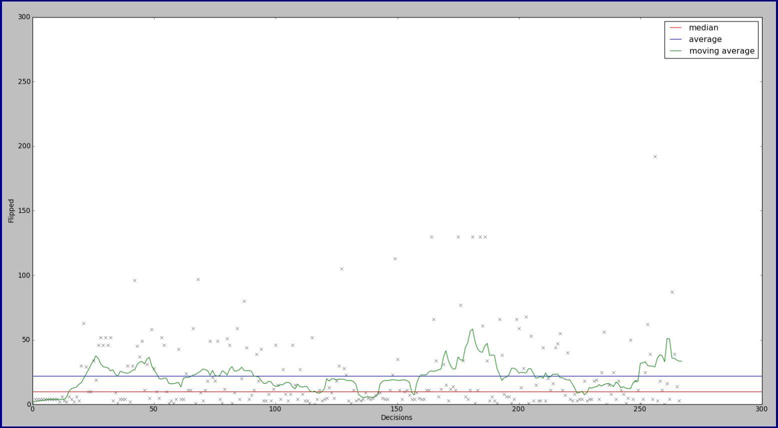
Visualization Examples

Towers of Hanoi: program interaction graph
Colors showing flipped assignments



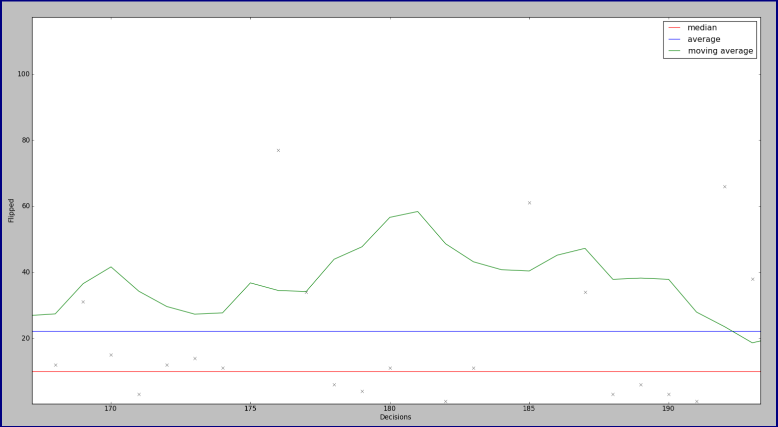
Visualization Examples

Towers of Hanoi: flipped assignments between decisions



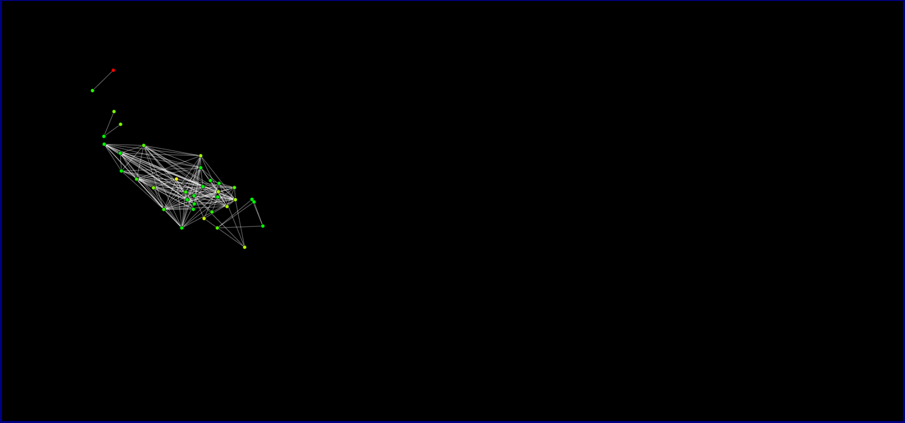
Visualization Examples

Towers of Hanoi: flipped assignments between decisions (zoomed in)



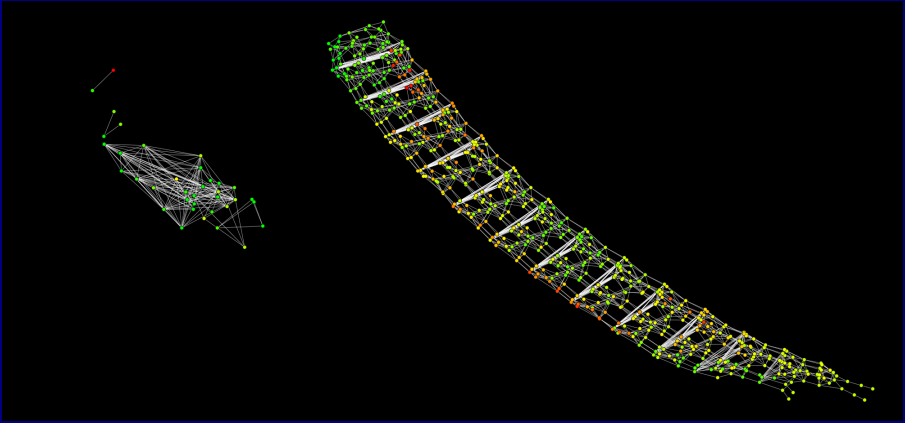
Visualization Examples

Towers of Hanoi: learned nogoods during zoomed in segment projected onto program interaction graph layout

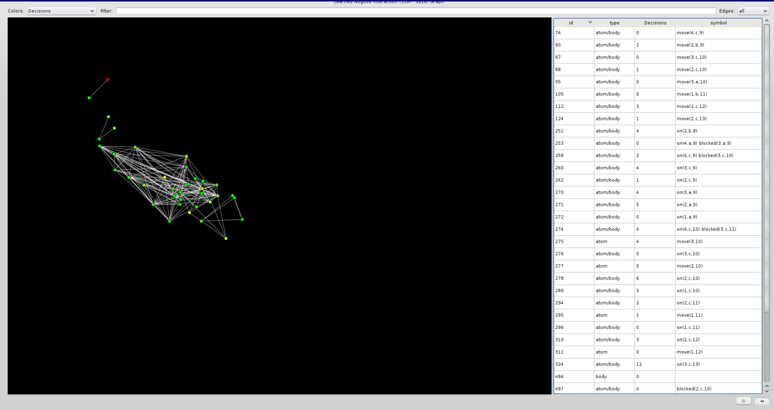


Visualization Examples

Towers of Hanoi: learned nogoods during zoomed in segment compared to program interaction graph



Interactive View



id	type	Decisions	symbol
74	atom/body	0	move(4,c,9)
80	atom/body	2	move(2,b,9)
87	atom/body	0	move(3,c,10)
88	atom/body	1	move(2,c,10)
95	atom/body	0	move(3,a,10)
105	atom/body	0	move(1,b,11)
112	atom/body	3	move(2,c,12)
124	atom/body	1	move(2,c,13)
251	atom/body	4	on(2,b,8)
253	atom/body	0	on(4,a,8) blocked(3,a,9)
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274	atom/body	4	on(4,c,10) blocked(3,c,11)
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278	atom/body	6	on(2,c,10)
280	atom/body	3	on(1,c,10)
294	atom/body	2	on(2,c,11)
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296	atom/body	0	on(1,c,11)
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311	atom	0	move(1,12)
324	atom/body	12	on(3,c,13)
484	body	0	
487	atom/body	0	blocked(2,c,10)

- Symbol table shows additional information about variables
- Search bar and symbol table allow for dynamic change of the view

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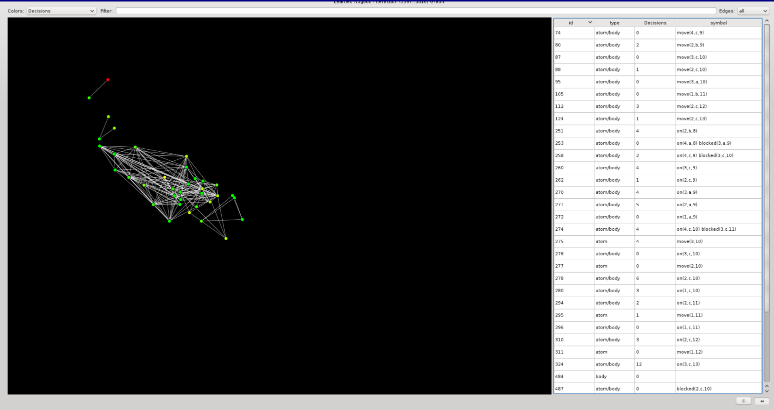

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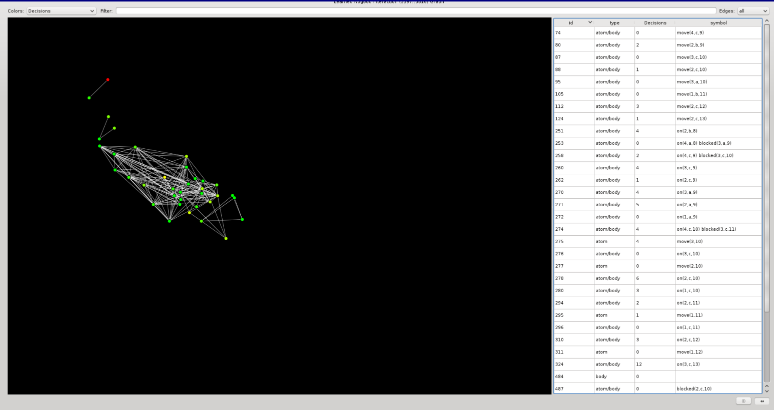

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Advanced Modeling: Overview

Anything left to worry about?

- ASP offers
  - rich yet easy modeling languages
  - efficient instantiation procedures
  - powerful search engines

- BUT The problem encoding (still) matters!

- Example Sort a list with 8 elements
  - divide-and-conquer $\sim$ $8(\log_2 8) = 16$ "operations"
  - permutation guessing $\sim$ $8!/2 = 20160$ "operations"

Potassco

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Outline

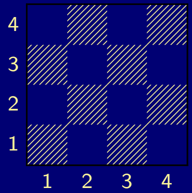
N-Queens Problem

Problem Specification

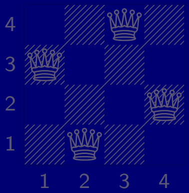
Given an $N \times N$ chessboard,
place N queens such that they do not attack each other
(neither horizontally, vertically, nor diagonally)

$N = 4$

Chessboard



Placement



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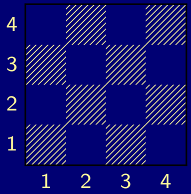

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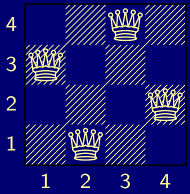

A First Encoding

1. Each square may host a queen
2. No row, column, or diagonal hosts two queens
3. A placement is given by instances of `queen` in a stable model

queens_0.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- queen(X1,Y1), queen(X1,Y2), Y1 < Y2.

% DISPLAY
#hide. #show queen/2.
```

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Anything missing?

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Potassco

A First Encoding

1. Each square may host a queen
2. No row, column, or diagonal hosts two queens
3. A placement is given by instances of `queen` in a stable model
4. We have to place (at least) N queens

queens_0.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
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% TEST
[...]
:- not n #count{ queen(X,Y) }.

% DISPLAY
#hide. #show queen/2.
```

A First Encoding

Let's Place 8 Queens!

```
gringo -c n=8 queens_0.lp | clasp --stats
```

```
Answer: 1
queen(1,6) queen(2,3) queen(3,1) queen(4,7)
queen(5,5) queen(6,8) queen(7,2) queen(8,4)
SATISFIABLE

Models      : 1+
Time        : 0.006s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
Choices     : 18
Conflicts   : 13
Restarts    : 0

Variables   : 793
Constraints : 729
```

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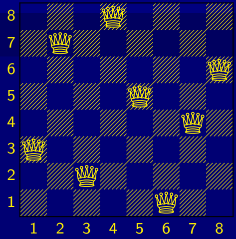


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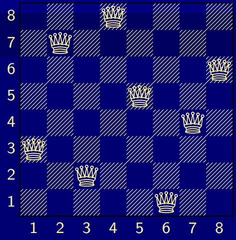


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A First Encoding

Let's Place 22 Queens!

```
gringo -c n=22 queens_0.lp | clasp --stats
```

```
Answer: 1
queen(1,10) queen(2,6) queen(3,16) queen(4,14) queen(5,8) ...
SATISFIABLE

Models      : 1+
Time        : 150.531s (Solving: 150.37s 1st Model: 150.34s Unsat: 0.00s)
CPU Time    : 147.480s
Choices     : 594960
Conflicts   : 574565
Restarts    : 19

Variables   : 17271
Constraints : 16787
```

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Let's Place 22 Queens!

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gringo -c n=22 queens_0.lp | clasp --stats
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Answer: 1
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A First Refinement

At least *N* queens? Exactly one queen per row and column!

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At least *N* queens?

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#const n=4. square(1..n,1..n).

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% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.
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A First Refinement

At least *N* queens? Exactly one queen per row and column!

queens_1.lp

```
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% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.


% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Let's Place 22 Queens!

```
gringo -c n=22 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,18) queen(2,10) queen(3,21) queen(4,3) queen(5,5) ...
SATISFIABLE

Models      : 1+
Time        : 0.113s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.020s
Choices     : 132
Conflicts   : 105
Restarts    : 1

Variables   : 7238
Constraints : 6710
```

A First Refinement

Let's Place 22 Queens!

```
gringo -c n=22 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,18) queen(2,10) queen(3,21) queen(4,3) queen(5,5) ...
SATISFIABLE

Models       : 1+
Time         : 0.113s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time     : 0.020s
Choices      : 132
Conflicts    : 105
Restarts     : 1

Variables    : 7238
Constraints  : 6710
```

Potassco

A First Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,24) queen(2,52) queen(3,37) queen(4,60) queen(5,76) ...
SATISFIABLE

Models      : 1+
Time        : 79.475s (Solving: 1.06s 1st Model: 1.06s Unsat: 0.00s)
CPU Time    : 6.930s
Choices     : 1373
Conflicts   : 845
Restarts    : 4

Variables   : 1211338
Constraints : 1196210
```

A First Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,24) queen(2,52) queen(3,37) queen(4,60) queen(5,76) ...
SATISFIABLE

Models      : 1+
Time        : 79.475s (Solving: 1.06s 1st Model: 1.06s Unsat: 0.00s)
CPU Time    : 6.930s
Choices     : 1373
Conflicts   : 845
Restarts    : 4

Variables   : 1211338
Constraints : 1196210
```

A First Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_1.lp | clasp --stats
```

```
Answer: 1
queen(1,24) queen(2,52) queen(3,37) queen(4,60) queen(5,76) ...
SATISFIABLE

Models      : 1+
Time        : 79.475s (Solving: 1.06s 1st Model: 1.06s Unsat: 0.00s)
CPU Time    : 6.930s
Choices     : 1373
Conflicts   : 845
Restarts    : 4

Variables   : 1211338
Constraints : 1196210
```

A First Refinement

Where Time Has Gone

```
time(gringo -c n=122 queens_1.lp | clasp --stats

1241358 7402724 24950848

real 1m15.468s
user 1m15.980s
sys  0m0.090s
```

Potassco

A First Refinement

Where Time Has Gone

```
time(gringo -c n=122 queens_1.lp | wc)
```

```
1241358 7402724 24950848

real 1m15.468s
user 1m15.980s
sys  0m0.090s
```

A First Refinement

Where Time Has Gone

```
time(gringo -c n=122 queens_1.lp | wc)
```

```
1241358 7402724 24950848

real  1m15.468s
user  1m15.980s
sys   0m0.090s
```

A First Refinement

Where Time Has Gone

```
time(gringo -c n=122 queens_1.lp | wc)
```

```
1241358 7402724 24950848

real 1m15.468s
user 1m15.980s
sys  0m0.090s
```

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                          O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                          O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                             O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                             O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.          O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

Potassco

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                   O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                   O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.     O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

Potassco

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                          O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                          O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                             O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                             O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.          O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                        O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                        O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                           O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                           O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.             O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                  O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                  O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                     O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                     O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.       O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time $\sim$ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                      O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                      O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                         O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                         O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.           O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time ~ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                        O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                        O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                           O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                           O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.         O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                     O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                     O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                        O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                        O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.          O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                    O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                    O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                       O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                       O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.         O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                        O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                        O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                           O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                           O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.           O(n^2×n^2)

% DISPLAY
#hide. #show queen/2.
```

Potassco

A First Refinement

Grounding Time ∼ Space

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).                                  O(n×n)

% GENERATE
{ queen(X,Y) } :- square(X,Y).                                  O(n×n)

% TEST
:- X := 1..n, not 1 #count{ queen(X,Y) } 1.                     O(n×n)
:- Y := 1..n, not 1 #count{ queen(X,Y) } 1.                     O(n×n)
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.       O(n^2×n^2)

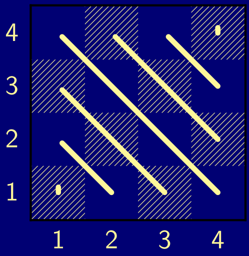
% DISPLAY
#hide. #show queen/2.
```

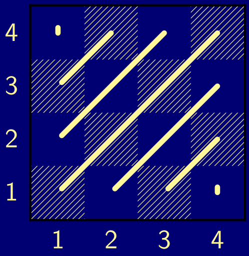
Diagonals make trouble!

Potassco

Enumerating Diagonals

$N = 4$





$\#\text{diagonal}_1 = (\#\text{row} + \#\text{column}) - 1$

$\#\text{diagonal}_2 = (\#\text{row} - \#\text{column}) + N$

- Note For each N, indexes $1, \ldots, (2*N)-1$ refer to squares on $\#\text{diagonal}_{1/2}$

Enumerating Diagonals

$N = 4$

4	4	5	6	7
3	3	4	5	6
2	2	3	4	5
1	1	2	3	4
	1	2	3	4

4	7	6	5	4
3	6	5	4	3
2	5	4	3	2
1	4	3	2	1
	1	2	3	4

$\#\text{diagonal}_1 = (\#\text{row} + \#\text{column}) - 1$

$\#\text{diagonal}_2 = (\#\text{row} - \#\text{column}) + N$

- Note For each N, indexes $1, \ldots, (2{*}N){-}1$ refer to squares on $\#\text{diagonal}_{1/2}$

Enumerating Diagonals

$N = 4$

4	**4**	**5**	**6**	**7**
3	**3**	**4**	**5**	**6**
2	**2**	**3**	**4**	**5**
1	**1**	**2**	**3**	**4**
	1	2	3	4

$\#\text{diagonal}_1 =$
$(\#\text{row} + \#\text{column}) - 1$

4	**7**	**6**	**5**	**4**
3	**6**	**5**	**4**	**3**
2	**5**	**4**	**3**	**2**
1	**4**	**3**	**2**	**1**
	1	2	3	4

$\#\text{diagonal}_2 =$
$(\#\text{row} - \#\text{column}) + N$

- Note For each N, indexes $1, \ldots, (2*N)-1$ refer to squares on $\#\text{diagonal}_{1/2}$

Enumerating Diagonals

$N = 4$

4	**4**	**5**	**6**	**7**
3	**3**	**4**	**5**	**6**
2	**2**	**3**	**4**	**5**
1	**1**	**2**	**3**	**4**
	1	2	3	4

$\#\mathrm{diagonal}_1 =$
$(\#\mathrm{row} + \#\mathrm{column}) - 1$

4	**7**	**6**	**5**	**4**
3	**6**	**5**	**4**	**3**
2	**5**	**4**	**3**	**2**
1	**4**	**3**	**2**	**1**
	1	2	3	4

$\#\mathrm{diagonal}_2 =$
$(\#\mathrm{row} - \#\mathrm{column}) + N$

- Note For each N, indexes $1, \ldots, (2*N)-1$ refer to squares on $\#\mathrm{diagonal}_{1/2}$

A Second Refinement

Let's go for Diagonals!

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- queen(X1,Y1), queen(X2,Y2), X1 < X2, X2-X1 == |Y2-Y1|.


% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's go for Diagonals!

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1


% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's go for Diagonals!

queens_1.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's go for Diagonals!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Second Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...
SATISFIABLE

Models      : 1+
Time        : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)
CPU Time    : 0.210s
Choices     : 11036
Conflicts   : 499
Restarts    : 3

Variables   : 16098
Constraints : 970
```

A Second Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats

Answer: 1
queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...
SATISFIABLE

Models      : 1+
Time        : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)
CPU Time    : 0.210s
Choices     : 11036
Conflicts   : 499
Restarts    : 3

Variables   : 16098
Constraints : 970
```

A Second Refinement

Let's Place 122 Queens!

```
gringo -c n=122 queens_2.lp | clasp --stats

Answer: 1
queen(1,98) queen(2,54) queen(3,89) queen(4,83) queen(5,59) ...
SATISFIABLE

Models      : 1+
Time        : 2.211s (Solving: 0.13s 1st Model: 0.13s Unsat: 0.00s)
CPU Time    : 0.210s
Choices     : 11036
Conflicts   : 499
Restarts    : 3

Variables   : 16098
Constraints : 970
```

A Second Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 35.450s (Solving: 6.69s 1st Model: 6.68s Unsat: 0.00s)
CPU Time    : 7.250s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Second Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 35.450s (Solving: 6.69s 1st Model: 6.68s Unsat: 0.00s)
CPU Time    : 7.250s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Second Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_2.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 35.450s (Solving: 6.69s 1st Model: 6.68s Unsat: 0.00s)
CPU Time    : 7.250s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Third Refinement

Let's Precalculate Indexes!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Precalculate Indexes!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X+Y)-1 }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : D == (X-Y)+n }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Precalculate Indexes!

queens_2.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag1(X,Y,D) }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag2(X,Y,D) }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Precalculate Indexes!

queens_3.lp

```
% DOMAIN
#const n=4. square(1..n,1..n).
diag1(X,Y,(X+Y)-1) :- square(X,Y). diag2(X,Y,(X-Y)+n) :- square(X,Y).

% GENERATE
0 #count{ queen(X,Y) } 1 :- square(X,Y).

% TEST
:- X = 1..n, not 1 #count{ queen(X,Y) } 1.
:- Y = 1..n, not 1 #count{ queen(X,Y) } 1.
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag1(X,Y,D) }. % Diagonal 1
:- D = 1..2*n-1, 2 #count{ queen(X,Y) : diag2(X,Y,D) }. % Diagonal 2

% DISPLAY
#hide. #show queen/2.
```

A Third Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
CPU Time    : 7.320s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Third Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
CPU Time    : 7.320s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Third Refinement

Let's Place 300 Queens!

```
gringo -c n=300 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,62) queen(2,232) queen(3,176) queen(4,241) queen(5,207) ...
SATISFIABLE

Models      : 1+
Time        : 8.889s (Solving: 6.61s 1st Model: 6.60s Unsat: 0.00s)
CPU Time    : 7.320s
Choices     : 141445
Conflicts   : 7488
Restarts    : 9

Variables   : 92994
Constraints : 2394
```

A Third Refinement

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models      : 1+
Time        : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time    : 68.620s
Choices     : 869379
Conflicts   : 25746
Restarts    : 12

Variables   : 365994
Constraints : 4794
```

A Third Refinement

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
```

```
Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models      : 1+
Time        : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time    : 68.620s
Choices     : 869379
Conflicts   : 25746
Restarts    : 12

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models      : 1+
Time        : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time    : 68.620s
Choices     : 869379
Conflicts   : 25746
Restarts    : 12

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic

Answer: 1
queen(1,477) queen(2,365) queen(3,455) queen(4,470) queen(5,237) ...
SATISFIABLE

Models      : 1+
Time        : 76.798s (Solving: 65.81s 1st Model: 65.75s Unsat: 0.00s)
CPU Time    : 68.620s
Choices     : 869379
Conflicts   : 25746
Restarts    : 12

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,422) queen(2,458) queen(3,224) queen(4,408) queen(5,405) ...
SATISFIABLE

Models       : 1+
Time         : 37.454s (Solving: 26.38s 1st Model: 26.26s Unsat: 0.00s)
CPU Time     : 29.580s
Choices      : 961315
Conflicts    : 3222
Restarts     : 7

Variables    : 365994
Constraints  : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,422) queen(2,458) queen(3,224) queen(4,408) queen(5,405) ...
SATISFIABLE

Models      : 1+
Time        : 37.454s (Solving: 26.38s 1st Model: 26.26s Unsat: 0.00s)
CPU Time    : 29.580s
Choices     : 961315
Conflicts   : 3222
Restarts    : 7

Variables   : 365994
Constraints : 4794
```

A Case for Oracles

Let's Place 600 Queens!

```
gringo -c n=600 queens_3.lp | clasp --stats
--heuristic=vsids --trans-ext=dynamic
```

```
Answer: 1
queen(1,90) queen(2,452) queen(3,494) queen(4,145) queen(5,84) ...
SATISFIABLE

Models      : 1+
Time        : 22.654s (Solving: 10.53s 1st Model: 10.47s Unsat: 0.00s)
CPU Time    : 15.750s
Choices     : 1058729
Conflicts   : 2128
Restarts    : 6

Variables   : 403123
Constraints : 49636
```

Outline

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. check all properties explicitly ... obsolete if properties change
2. use variable-sized conjunction (via ':') ... adapts to changing facts
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap).   pro(cucumber,cheap).
pro(asparagus,fresh).   pro(cucumber,fresh).
pro(asparagus,tasty).   pro(cucumber,tasty).
```

Potassco

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. check all properties explicitly ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).        veg(cucumber).
pro(asparagus,cheap). pro(cucumber,cheap).
pro(asparagus,fresh). pro(cucumber,fresh).
pro(asparagus,tasty). pro(cucumber,tasty).
```

```
buy(X) :- veg(X), pro(X,cheap), pro(X,fresh), pro(X,tasty).
```

Potassco

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. check all properties explicitly ... obsolete if properties change
2. use variable-sized conjunction (via ':') ... adapts to changing facts
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).           veg(cucumber).
pro(asparagus,cheap).     pro(cucumber,cheap).
pro(asparagus,fresh).     pro(cucumber,fresh).
pro(asparagus,tasty).     pro(cucumber,tasty).
pro(asparagus,clean).

buy(X) :- veg(X), pro(X,cheap), pro(X,fresh), pro(X,tasty), pro(X,clean).
```

Potassco

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✓
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap).   pro(cucumber,cheap).
pro(asparagus,fresh).   pro(cucumber,fresh).
pro(asparagus,tasty).   pro(cucumber,tasty).
pro(asparagus,clean).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts

Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap).  pro(cucumber,cheap).  pre(cheap).
pro(asparagus,fresh).  pro(cucumber,fresh).  pre(fresh).
pro(asparagus,tasty).  pro(cucumber,tasty).  pre(tasty).
pro(asparagus,clean).

buy(X) :- veg(X), pro(X,P) : pre(P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✓
3. use negation of complement ... adapts to changing facts ✓

Example: vegetables to buy

```
veg(asparagus).          veg(cucumber).
pro(asparagus,cheap).    pro(cucumber,cheap).    pre(cheap).
pro(asparagus,fresh).    pro(cucumber,fresh).    pre(fresh).
pro(asparagus,tasty).    pro(cucumber,tasty).    pre(tasty).
pro(asparagus,clean).                            pre(clean).

buy(X) :- veg(X), pro(X,P) : pre(P).
```

Potassco

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ . . . obsolete if properties change ✗
2. use variable-sized conjunction (via ':') . . . adapts to changing facts ✔
3. use negation of complement . . . adapts to changing facts ✔

Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap). pro(cucumber,cheap). pre(cheap).
pro(asparagus,fresh). pro(cucumber,fresh). pre(fresh).
pro(asparagus,tasty). pro(cucumber,tasty). pre(tasty).
pro(asparagus,clean).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts ✔

Example: vegetables to buy

```
veg(asparagus).        veg(cucumber).
pro(asparagus,cheap). pro(cucumber,cheap). pre(cheap).
pro(asparagus,fresh). pro(cucumber,fresh). pre(fresh).
pro(asparagus,tasty). pro(cucumber,tasty). pre(tasty).
pro(asparagus,clean).

buy(X) :- veg(X), not bye(X).     bye(X) :- veg(X), pre(P), not pro(X,P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✗
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts ✔

Example: vegetables to buy

```
veg(asparagus).         veg(cucumber).
pro(asparagus,cheap).   pro(cucumber,cheap).   pre(cheap).
pro(asparagus,fresh).   pro(cucumber,fresh).   pre(fresh).
pro(asparagus,tasty).   pro(cucumber,tasty).   pre(tasty).
pro(asparagus,clean).                          pre(clean).

buy(X) :- veg(X), not bye(X).     bye(X) :- veg(X), pre(P), not pro(X,P).
```

Implementing Universal Quantification

Goal: identify objects such that ALL properties from a "list" hold

1. ~~check all properties explicitly~~ ... obsolete if properties change ✘
2. use variable-sized conjunction (via ':') ... adapts to changing facts ✔
3. use negation of complement ... adapts to changing facts ✔

Example: vegetables to buy

```
veg(asparagus).        veg(cucumber).
pro(asparagus,cheap). pro(cucumber,cheap). pre(cheap).
pro(asparagus,fresh). pro(cucumber,fresh). pre(fresh).
pro(asparagus,tasty). pro(cucumber,tasty). pre(tasty).
pro(asparagus,clean).                      pre(clean).

buy(X) :- veg(X), not bye(X).     bye(X) :- veg(X), pre(P), not pro(X,P).
```

Potassco

Running Example: Latin Square

Given: an $N \times N$ board

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

represented by facts:

```
square(1,1). ... square(1,6).
square(2,1). ... square(2,6).
square(3,1). ... square(3,6).
square(4,1). ... square(4,6).
square(5,1). ... square(5,6).
square(6,1). ... square(6,6).
```

Wanted: assignment of $1, \ldots, N$

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	3	4	5	6	1
3	3	4	5	6	1	2
4	4	5	6	1	2	3
5	5	6	1	2	3	4
6	6	1	2	3	4	5

represented by atoms:

```
num(1,1,1) num(1,2,2) ... num(1,6,6)
num(2,1,2) num(2,2,3) ... num(2,6,1)
num(3,1,3) num(3,2,4) ... num(3,6,2)
num(4,1,4) num(4,2,5) ... num(4,6,3)
num(5,1,5) num(5,2,6) ... num(5,6,4)
num(6,1,6) num(6,2,1) ... num(6,6.5)
```

Running Example: Latin Square

Given: an $N \times N$ board

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

represented by facts:

```
square(1,1). ... square(1,6).
square(2,1). ... square(2,6).
square(3,1). ... square(3,6).
square(4,1). ... square(4,6).
square(5,1). ... square(5,6).
square(6,1). ... square(6,6).
```

Wanted: assignment of $1, \ldots, N$

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	3	4	5	6	1
3	3	4	5	6	1	2
4	4	5	6	1	2	3
5	5	6	1	2	3	4
6	6	1	2	3	4	5

represented by atoms:

```
num(1,1,1) num(1,2,2) ... num(1,6,6)
num(2,1,2) num(2,2,3) ... num(2,6,1)
num(3,1,3) num(3,2,4) ... num(3,6,2)
num(4,1,4) num(4,2,5) ... num(4,6,3)
num(5,1,5) num(5,2,6) ... num(5,6,4)
num(6,1,6) num(6,2,1) ... num(6,6.5)
```

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).



% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- square(X1,Y1), N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- square(X1,Y1), N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

- Note unreused "singleton variables"

```
gringo latin_0.lp | wc
```

105480 2558984 14005258

```
gringo latin_1.lp | wc
```

42056 273672 1690522

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).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```

```
gringo latin_1.lp | wc

42056 273672 1690522
```

Projecting Irrelevant Details Out

A Latin square encoding

```
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#const n=32. square(1..n,1..n).



% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- square(X1,Y1), N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- square(X1,Y1), N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

- Note unreused "singleton variables"

```
gringo latin_0.lp | wc
```

105480 2558984 14005258

```
gringo latin_1.lp | wc
```

42056 273672 1690522

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
squareX(X) :- square(X,Y).      squareY(Y) :- square(X,Y).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- squareX(X1)  , N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- squareY(Y1)  , N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

- Note unreused "singleton variables"

```
gringo latin_0.lp | wc
```

105480 2558984 14005258

```
gringo latin_1.lp | wc
```

42056 273672 1690522

Projecting Irrelevant Details Out

A Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
squareX(X) :- square(X,Y).      squareY(Y) :- square(X,Y).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- squareX(X1)  , N = 1..n, not num(X1,Y2,N) : square(X1,Y2).
:- squareY(Y1)  , N = 1..n, not num(X2,Y1,N) : square(X2,Y1).
```

- Note unreused "singleton variables"

```
gringo latin_0.lp | wc
```

105480 2558984 14005258

```
gringo latin_1.lp | wc
```

42056 273672 1690522

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 != Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 != X2.
```

- Note duplicate ground rules (swapping `Y1`/`Y2` and `X1`/`X2` gives the "same")

```
gringo latin_2.lp | wc
```

2071560 12389384 40906946

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 != Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 != X2.
```

- Note duplicate ground rules (swapping Y1/Y2 and X1/X2 gives the "same")

```
gringo latin_2.lp | wc
```

2071560 12389384 40906946

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 != Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 != X2.
```

- Note duplicate ground rules (swapping `Y1`/`Y2` and `X1`/`X2` gives the "same")

```
gringo latin_2.lp | wc
```

2071560 12389384 40906946

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 <  Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 <  X2.
```

- Note duplicate ground rules (swapping Y1/Y2 and X1/X2 gives the "same")

```
gringo latin_2.lp | wc
```

2071560 12389384 40906946

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

Unraveling Symmetric Inequalities

Another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 <  Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 <  X2.
```

- Note duplicate ground rules (swapping `Y1`/`Y2` and `X1`/`X2` gives the "same")

```
gringo latin_2.lp | wc
```

2071560 12389384 40906946

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 < Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 < X2.
```

- Note uniqueness of `N` in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 < Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 < X2.
```

- Note uniqueness of `N` in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

```
1055752 6294536 21099558
```

```
gringo latin_4.lp | wc
```

```
228360 1205256 4780744
```

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- num(X1,Y1,N), num(X1,Y2,N), Y1 < Y2.
:- num(X1,Y1,N), num(X2,Y1,N), X1 < X2.
```

- Note uniqueness of `N` in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
 :- num(X,Y,N), gtX(X,Y,N).              :- num(X,Y,N), gtY(X,Y,N).
```

- Note uniqueness of `N` in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
 :- num(X,Y,N), gtX(X,Y,N).              :- num(X,Y,N), gtY(X,Y,N).
```

- Note uniqueness of `N` in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

Linearizing Existence Tests

Still another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
gtX(X-1,Y,N) :- num(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- num(X,Y,N), 1 < Y.
gtX(X-1,Y,N) :- gtX(X,Y,N), 1 < X.      gtY(X,Y-1,N) :- gtY(X,Y,N), 1 < Y.
 :- num(X,Y,N), gtX(X,Y,N).              :- num(X,Y,N), gtY(X,Y,N).
```

- Note uniqueness of `N` in a row/column checked by ENUMERATING PAIRS!

```
gringo latin_3.lp | wc
```

1055752 6294536 21099558

```
gringo latin_4.lp | wc
```

228360 1205256 4780744

Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

```
gringo latin_5.lp | wc
304136 5778440 30252505
```

```
gringo latin_6.lp | wc
48136 373768 2185042
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Assigning Aggregate Values

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Assigning Aggregate Values

Yet another Latin square encoding

```
% DOMAIN
#const n=32. square(1..n,1..n).
sigma(S) :- S = #sum[ square(X,n) = X ].                         ✔

% GENERATE
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occX(X,N,C) :- X = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.
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% DEFINE + TEST
occX(X,N,C) :- X = 1..n, N = 1..n, C #count{ num(X,Y,N) } C, C = 0..n.
occY(Y,N,C) :- Y = 1..n, N = 1..n, C #count{ num(X,Y,N) } C, C = 0..n.
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

% DISPLAY
#hide. #show num/3. #show sigma/1.
```

- Note internal transformation by gringo

Assigning Aggregate Values

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occY(Y,N,C) :- Y = 1..n, N = 1..n, C = #count{ num(X,Y,N) }.     ✗
:- occX(X,N,C), C != 1.  :- occY(Y,N,C), C != 1.

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% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
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Breaking Symmetries

The ultimate Latin square encoding?

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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

- Note many symmetric solutions (mirroring, rotation, value permutation)

Breaking Symmetries

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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.



% DISPLAY
#hide. #show num/3.
```

- Note easy and safe to fix a full row/column!

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% DOMAIN
#const n=32. square(1..n,1..n).

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1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

- Note easy and safe to fix a full row/column!

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#const n=32. square(1..n,1..n).

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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

- Note Let's compare enumeration speed!

Breaking Symmetries

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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_6.lp | clasp -q 0
```

Breaking Symmetries

The ultimate Latin square encoding?

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#const n=32. square(1..n,1..n).

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1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

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:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.


% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_6.lp | clasp -q 0
```

```
Models : 161280     Time : 2.078s
```

Breaking Symmetries

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```
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#const n=32. square(1..n,1..n).

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1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

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:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_7.lp | clasp -q 0
```

```
Models : 161280    Time : 2.078s
```

Breaking Symmetries

The ultimate Latin square encoding?

```
% DOMAIN
#const n=32. square(1..n,1..n).

% GENERATE
1 #count{ num(X,Y,N) : N = 1..n } 1 :- square(X,Y).

% TEST
:- X = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- Y = 1..n, N = 1..n, not 1 #count{ num(X,Y,N) } 1.
:- square(1,Y), not num(1,Y,Y). % Symmetry Breaking

% DISPLAY
#hide. #show num/3.
```

```
gringo -c n=5 latin_7.lp | clasp -q 0
```

```
Models : 1344       Time : 0.024s
```

Outline

Encode With Care!

1. Create a working encoding
   - Q1: Do you need to modify the encoding if the facts change?
   - Q2: Are all variables significant (or statically functionally dependent)?
   - Q3: Can there be (many) identic ground rules?
   - Q4: Do you enumerate pairs of values (to test uniqueness)?
   - Q5: Do you assign dynamic aggregate values (to check a fixed bound)?
   - Q6: Do you admit (obvious) symmetric solutions?
   - Q7: Do you have additional domain knowledge simplifying the problem?
   - Q8: Are you aware of anything else that, if encoded, would reduce grounding and/or solving efforts?
2. Revise until no "Yes" answer!
   - Note If the format of facts makes encoding painful (for instance, abusing grounding for "scientific calculations"), revise the fact format as well.

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Some Hints on (Preventing) Debugging

Kinds of errors

- syntactic . . . follow error messages by the grounder
- semantic . . . (most likely) encoding/intention mismatch

Ways to identify semantic errors (early)

- develop and test incrementally
  - prepare toy instances with "interesting features"
  - build the encoding bottom-up and verify additions (eg. new predicates)
- compare the encoded to the intended meaning
  - check whether the grounding fits (use `gringo -t`)
  - if stable models are unintended, investigate conditions that fail to hold
  - if stable models are missing, examine integrity constraints (add heads)
- ask tools (eg. `http://www.kr.tuwien.ac.at/research/projects/mmdasp/`)

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Overcoming Performance Bottlenecks

Grounding

- monitor time spent by and output size of `gringo`
  1. system tools (eg. `time(gringo [...] | wc)`)
  2. profiling info (eg. `gringo --gstats --verbose=3 [...] > /dev/null`)
- Note once identified, reformulate "critical" logic program parts

Solving

- check solving statistics (use `clasp --stats`)

  if great search efforts (`Conflicts`/`Choices`/`Restarts`), then

  try prefabricated settings (using `clasp` option '`--configuration`')

  try auto-configuration (offered by `claspfolio`)

  try manual fine-tuning (requires expert knowledge!)

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  1. try prefabricated settings (using `clasp` option '`--configuration`')
  2. try auto-configuration (offered by `claspfolio`)
  3. try manual fine-tuning (requires expert knowledge!)
  4. if possible, reformulate the problem or add domain knowledge ("redundant" constraints) to help the solver

Overcoming Performance Bottlenecks

Grounding

- monitor time spent by and output size of `gringo`
  1. system tools (eg. `time(gringo [...] | wc)`)
  2. profiling info (eg. `gringo --gstats --verbose=3 [...] > /dev/null`)
- Note once identified, reformulate "critical" logic program parts

Solving

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Outline

Summary

- ASP is a viable tool for Knowledge Representation and Reasoning
- ASP offers efficient and versatile off-the-shelf solving technology
- ASP offers an expanding functionality and ease of use
    - Rapid application development tool
- ASP has a growing range of applications

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`http://potassco.sourceforge.net`

[1] C. Anger, M. Gebser, T. Linke, A. Neumann, and T. Schaub.
The `nomore++` approach to answer set solving.
In G. Sutcliffe and A. Voronkov, editors, *Proceedings of the Twelfth International Conference on Logic for Programming, Artificial Intelligence, and Reasoning (LPAR'05)*, volume 3835 of *Lecture Notes in Artificial Intelligence*, pages 95–109. Springer-Verlag, 2005.

[2] C. Anger, K. Konczak, T. Linke, and T. Schaub.
A glimpse of answer set programming.
Künstliche Intelligenz, 19(1):12–17, 2005.

[3] Y. Babovich and V. Lifschitz.
Computing answer sets using program completion.
Unpublished draft, 2003.

[4] C. Baral.
Knowledge Representation, Reasoning and Declarative Problem Solving.
Cambridge University Press, 2003.

[5] C. Baral, G. Brewka, and J. Schlipf, editors.
Proceedings of the Ninth International Conference on Logic Programming and Nonmonotonic Reasoning (LPNMR'07), volume 4483 of *Lecture Notes in Artificial Intelligence*. Springer-Verlag, 2007.

[6] C. Baral and M. Gelfond.
Logic programming and knowledge representation.
Journal of Logic Programming, 12:1–80, 1994.

[7] S. Baselice, P. Bonatti, and M. Gelfond.
Towards an integration of answer set and constraint solving.
In M. Gabbrielli and G. Gupta, editors, *Proceedings of the Twenty-first International Conference on Logic Programming (ICLP'05)*, volume 3668 of *Lecture Notes in Computer Science*, pages 52–66. Springer-Verlag, 2005.

[8] A. Biere.
Adaptive restart strategies for conflict driven SAT solvers.

In H. Kleine Büning and X. Zhao, editors, *Proceedings of the Eleventh International Conference on Theory and Applications of Satisfiability Testing (SAT'08)*, volume 4996 of *Lecture Notes in Computer Science*, pages 28–33. Springer-Verlag, 2008.

[9] A. Biere.
PicoSAT essentials.
Journal on Satisfiability, Boolean Modeling and Computation, 4:75–97, 2008.

[10] A. Biere, M. Heule, H. van Maaren, and T. Walsh, editors.
Handbook of Satisfiability, volume 185 of *Frontiers in Artificial Intelligence and Applications*.
IOS Press, 2009.

[11] G. Brewka, T. Eiter, and M. Truszczyński.
Answer set programming at a glance.
Communications of the ACM, 54(12):92–103, 2011.

[12] K. Clark.
Negation as failure.

In H. Gallaire and J. Minker, editors, *Logic and Data Bases*, pages 293–322. Plenum Press, 1978.

[13] M. D'Agostino, D. Gabbay, R. Hähnle, and J. Posegga, editors.
Handbook of Tableau Methods.
Kluwer Academic Publishers, 1999.

[14] E. Dantsin, T. Eiter, G. Gottlob, and A. Voronkov.
Complexity and expressive power of logic programming.
In *Proceedings of the Twelfth Annual IEEE Conference on Computational Complexity (CCC'97)*, pages 82–101. IEEE Computer Society Press, 1997.

[15] M. Davis, G. Logemann, and D. Loveland.
A machine program for theorem-proving.
Communications of the ACM, 5:394–397, 1962.

[16] M. Davis and H. Putnam.
A computing procedure for quantification theory.
Journal of the ACM, 7:201–215, 1960.

Potassco

[17] C. Drescher, M. Gebser, T. Grote, B. Kaufmann, A. König, M. Ostrowski, and T. Schaub.
Conflict-driven disjunctive answer set solving.
In G. Brewka and J. Lang, editors, *Proceedings of the Eleventh International Conference on Principles of Knowledge Representation and Reasoning (KR'08)*, pages 422–432. AAAI Press, 2008.

[18] C. Drescher, M. Gebser, B. Kaufmann, and T. Schaub.
Heuristics in conflict resolution.
In M. Pagnucco and M. Thielscher, editors, *Proceedings of the Twelfth International Workshop on Nonmonotonic Reasoning (NMR'08)*, number UNSW-CSE-TR-0819 in School of Computer Science and Engineering, The University of New South Wales, Technical Report Series, pages 141–149, 2008.

[19] N. Eén and N. Sörensson.
An extensible SAT-solver.
In E. Giunchiglia and A. Tacchella, editors, *Proceedings of the Sixth International Conference on Theory and Applications of Satisfiability*

Testing (SAT'03), volume 2919 of *Lecture Notes in Computer Science*, pages 502–518. Springer-Verlag, 2004.

[20] T. Eiter and G. Gottlob.
On the computational cost of disjunctive logic programming: Propositional case.
Annals of Mathematics and Artificial Intelligence, 15(3-4):289–323, 1995.

[21] T. Eiter, G. Ianni, and T. Krennwallner.
Answer Set Programming: A Primer.
In S. Tessaris, E. Franconi, T. Eiter, C. Gutierrez, S. Handschuh, M. Rousset, and R. Schmidt, editors, *Fifth International Reasoning Web Summer School (RW'09)*, volume 5689 of *Lecture Notes in Computer Science*, pages 40–110. Springer-Verlag, 2009.

[22] F. Fages.
Consistency of Clark's completion and the existence of stable models.
Journal of Methods of Logic in Computer Science, 1:51–60, 1994.

[23] P. Ferraris.

Answer sets for propositional theories.
In C. Baral, G. Greco, N. Leone, and G. Terracina, editors, *Proceedings of the Eighth International Conference on Logic Programming and Nonmonotonic Reasoning (LPNMR'05)*, volume 3662 of *Lecture Notes in Artificial Intelligence*, pages 119–131. Springer-Verlag, 2005.

[24] P. Ferraris and V. Lifschitz.
Mathematical foundations of answer set programming.
In S. Artëmov, H. Barringer, A. d'Avila Garcez, L. Lamb, and J. Woods, editors, *We Will Show Them! Essays in Honour of Dov Gabbay*, volume 1, pages 615–664. College Publications, 2005.

[25] M. Fitting.
A Kripke-Kleene semantics for logic programs.
Journal of Logic Programming, 2(4):295–312, 1985.

[26] M. Gebser, R. Kaminski, B. Kaufmann, M. Ostrowski, T. Schaub, and S. Thiele.
A user's guide to `gringo`, `clasp`, `clingo`, and `iclingo`.

[27] M. Gebser, R. Kaminski, B. Kaufmann, M. Ostrowski, T. Schaub, and S. Thiele.
Engineering an incremental ASP solver.
In M. Garcia de la Banda and E. Pontelli, editors, *Proceedings of the Twenty-fourth International Conference on Logic Programming (ICLP'08)*, volume 5366 of *Lecture Notes in Computer Science*, pages 190–205. Springer-Verlag, 2008.

[28] M. Gebser, R. Kaminski, B. Kaufmann, and T. Schaub.
On the implementation of weight constraint rules in conflict-driven ASP solvers.
In Hill and Warren [44], pages 250–264.

[29] M. Gebser, R. Kaminski, B. Kaufmann, and T. Schaub.
Answer Set Solving in Practice.
Synthesis Lectures on Artificial Intelligence and Machine Learning. Morgan and Claypool Publishers, 2012.

[30] M. Gebser, B. Kaufmann, A. Neumann, and T. Schaub.

clasp: A conflict-driven answer set solver.
In Baral et al. [5], pages 260–265.

[31] M. Gebser, B. Kaufmann, A. Neumann, and T. Schaub.
Conflict-driven answer set enumeration.
In Baral et al. [5], pages 136–148.

[32] M. Gebser, B. Kaufmann, A. Neumann, and T. Schaub.
Conflict-driven answer set solving.
In Veloso [68], pages 386–392.

[33] M. Gebser, B. Kaufmann, A. Neumann, and T. Schaub.
Advanced preprocessing for answer set solving.
In M. Ghallab, C. Spyropoulos, N. Fakotakis, and N. Avouris, editors, *Proceedings of the Eighteenth European Conference on Artificial Intelligence (ECAI'08)*, pages 15–19. IOS Press, 2008.

[34] M. Gebser, B. Kaufmann, and T. Schaub.
The conflict-driven answer set solver clasp: Progress report.

In E. Erdem, F. Lin, and T. Schaub, editors, *Proceedings of the Tenth International Conference on Logic Programming and Nonmonotonic Reasoning (LPNMR'09)*, volume 5753 of *Lecture Notes in Artificial Intelligence*, pages 509–514. Springer-Verlag, 2009.

[35] M. Gebser, B. Kaufmann, and T. Schaub.
Solution enumeration for projected Boolean search problems.
In W. van Hoeve and J. Hooker, editors, *Proceedings of the Sixth International Conference on Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems (CPAIOR'09)*, volume 5547 of *Lecture Notes in Computer Science*, pages 71–86. Springer-Verlag, 2009.

[36] M. Gebser, M. Ostrowski, and T. Schaub.
Constraint answer set solving.
In Hill and Warren [44], pages 235–249.

[37] M. Gebser and T. Schaub.
Tableau calculi for answer set programming.

Potassco

In S. Etalle and M. Truszczyński, editors, *Proceedings of the Twenty-second International Conference on Logic Programming (ICLP'06)*, volume 4079 of *Lecture Notes in Computer Science*, pages 11–25. Springer-Verlag, 2006.

[38] M. Gebser and T. Schaub.
Generic tableaux for answer set programming.
In V. Dahl and I. Niemelä, editors, *Proceedings of the Twenty-third International Conference on Logic Programming (ICLP'07)*, volume 4670 of *Lecture Notes in Computer Science*, pages 119–133. Springer-Verlag, 2007.

[39] M. Gelfond.
Answer sets.
In V. Lifschitz, F. van Harmelen, and B. Porter, editors, *Handbook of Knowledge Representation*, chapter 7, pages 285–316. Elsevier Science, 2008.

[40] M. Gelfond and N. Leone.

Logic programming and knowledge representation — the A-Prolog perspective.
Artificial Intelligence, 138(1-2):3–38, 2002.

[41] M. Gelfond and V. Lifschitz.
The stable model semantics for logic programming.
In R. Kowalski and K. Bowen, editors, *Proceedings of the Fifth International Conference and Symposium of Logic Programming (ICLP'88)*, pages 1070–1080. MIT Press, 1988.

[42] M. Gelfond and V. Lifschitz.
Logic programs with classical negation.
In D. Warren and P. Szeredi, editors, *Proceedings of the Seventh International Conference on Logic Programming (ICLP'90)*, pages 579–597. MIT Press, 1990.

[43] E. Giunchiglia, Y. Lierler, and M. Maratea.
Answer set programming based on propositional satisfiability.
Journal of Automated Reasoning, 36(4):345–377, 2006.

[44] P. Hill and D. Warren, editors.
Proceedings of the Twenty-fifth International Conference on Logic Programming (ICLP'09), volume 5649 of *Lecture Notes in Computer Science*. Springer-Verlag, 2009.

[45] J. Huang.
The effect of restarts on the efficiency of clause learning.
In Veloso [68], pages 2318–2323.

[46] K. Konczak, T. Linke, and T. Schaub.
Graphs and colorings for answer set programming.
Theory and Practice of Logic Programming, 6(1-2):61–106, 2006.

[47] J. Lee.
A model-theoretic counterpart of loop formulas.
In L. Kaelbling and A. Saffiotti, editors, *Proceedings of the Nineteenth International Joint Conference on Artificial Intelligence (IJCAI'05)*, pages 503–508. Professional Book Center, 2005.

[48] N. Leone, G. Pfeifer, W. Faber, T. Eiter, G. Gottlob, S. Perri, and F. Scarcello.
The DLV system for knowledge representation and reasoning.
ACM Transactions on Computational Logic, 7(3):499–562, 2006.

[49] V. Lifschitz.
Answer set programming and plan generation.
Artificial Intelligence, 138(1-2):39–54, 2002.

[50] V. Lifschitz.
Introduction to answer set programming.
Unpublished draft, 2004.

[51] V. Lifschitz and A. Razborov.
Why are there so many loop formulas?
ACM Transactions on Computational Logic, 7(2):261–268, 2006.

[52] F. Lin and Y. Zhao.
ASSAT: computing answer sets of a logic program by SAT solvers.
Artificial Intelligence, 157(1-2):115–137, 2004.

[53] V. Marek and M. Truszczyński.
Nonmonotonic logic: context-dependent reasoning.
Artifical Intelligence. Springer-Verlag, 1993.

[54] V. Marek and M. Truszczyński.
Stable models and an alternative logic programming paradigm.
In K. Apt, V. Marek, M. Truszczyński, and D. Warren, editors, *The Logic Programming Paradigm: a 25-Year Perspective*, pages 375–398. Springer-Verlag, 1999.

[55] J. Marques-Silva, I. Lynce, and S. Malik.
Conflict-driven clause learning SAT solvers.
In Biere et al. [10], chapter 4, pages 131–153.

[56] J. Marques-Silva and K. Sakallah.
GRASP: A search algorithm for propositional satisfiability.
IEEE Transactions on Computers, 48(5):506–521, 1999.

[57] V. Mellarkod and M. Gelfond.
Integrating answer set reasoning with constraint solving techniques.

In J. Garrigue and M. Hermenegildo, editors, *Proceedings of the Ninth International Symposium on Functional and Logic Programming (FLOPS'08)*, volume 4989 of *Lecture Notes in Computer Science*, pages 15–31. Springer-Verlag, 2008.

[58] V. Mellarkod, M. Gelfond, and Y. Zhang.
Integrating answer set programming and constraint logic programming.
Annals of Mathematics and Artificial Intelligence, 53(1-4):251–287, 2008.

[59] D. Mitchell.
A SAT solver primer.
Bulletin of the European Association for Theoretical Computer Science, 85:112–133, 2005.

[60] M. Moskewicz, C. Madigan, Y. Zhao, L. Zhang, and S. Malik.
Chaff: Engineering an efficient SAT solver.
In *Proceedings of the Thirty-eighth Conference on Design Automation (DAC'01)*, pages 530–535. ACM Press, 2001.

[61] I. Niemelä.
Logic programs with stable model semantics as a constraint programming paradigm.
Annals of Mathematics and Artificial Intelligence, 25(3-4):241–273, 1999.

[62] R. Nieuwenhuis, A. Oliveras, and C. Tinelli.
Solving SAT and SAT modulo theories: From an abstract Davis-Putnam-Logemann-Loveland procedure to DPLL(T).
Journal of the ACM, 53(6):937–977, 2006.

[63] K. Pipatsrisawat and A. Darwiche.
A lightweight component caching scheme for satisfiability solvers.
In J. Marques-Silva and K. Sakallah, editors, *Proceedings of the Tenth International Conference on Theory and Applications of Satisfiability Testing (SAT'07)*, volume 4501 of *Lecture Notes in Computer Science*, pages 294–299. Springer-Verlag, 2007.

[64] L. Ryan.
Efficient algorithms for clause-learning SAT solvers.

Master's thesis, Simon Fraser University, 2004.

[65] P. Simons, I. Niemelä, and T. Soininen.
Extending and implementing the stable model semantics.
Artificial Intelligence, 138(1-2):181–234, 2002.

[66] T. Syrjänen.
Lparse 1.0 user's manual.

[67] A. Van Gelder, K. Ross, and J. Schlipf.
The well-founded semantics for general logic programs.
Journal of the ACM, 38(3):620–650, 1991.

[68] M. Veloso, editor.
Proceedings of the Twentieth International Joint Conference on Artificial Intelligence (IJCAI'07). AAAI/MIT Press, 2007.

[69] L. Zhang, C. Madigan, M. Moskewicz, and S. Malik.
Efficient conflict driven learning in a Boolean satisfiability solver.
In *Proceedings of the International Conference on Computer-Aided Design (ICCAD'01)*, pages 279–285. ACM Press, 2001.